CAUSAL AND GEOMETRIC RELATIONS*

Douglas W. Shrader, Jr.

Is space curved? The discovery of Non-Euclidean geometries inspired several attempts to determine empirically the topology of physical space. Hans Reichenbach recognized that a number of causal principles are presupposed in these experiments and suggested that, for geometry to be an empirical matter, we have to introduce the requirement that no causal relations are violated (that there are no causal anomalies). Though objectionable in certain respects, I think Reichenbach's thesis represents a significant insight into the interdependence of geometry and causation. The geometry we ascribe to physical space will help determine the nature of our causal laws, and vice-versa. But close as the connections between geometry and causation might be, they do not guarantee a unique prescription--even in conjunction with unlimited experience, a system of causal laws may not suffice to pick out a unique geometry, or vice-versa.

1. <u>Introduction</u>. Euclid developed his system of planar geometry sometime around 300 B.C. Centuries passed before anyone realized that other geometric systems were even mathematically possible. Today we have a well-accepted scientific theory (Einstein's General Theory of Relativity) which actually claims that space is curved (that a Non-Euclidean geometry applies).

But the choice of a geometry is not purely arbitrary. Well over a hundred years ago Gauss devised an experiment to trace a triangle between three mountain peaks so as to determine whether or not the interior angles summed to a number different from 180. Since then there have been a number of other attempts to determine empirically the topology of physical space (i.e. to decide which geometry applies).

Hans Reichenbach recognized that a number of causal principles are presupposed in these experiments and suggested that, for geometry to be an empirical matter, we have to introduce the requirement that no causal relations are violated (that there are no causal anomalies). Only in this way, says Reichenbach, does the question about the topology of space even constitute a well-determined question.[1]

Though objectionable in certain respects, I think Reichenbach's thesis represents a significant insight into the interdependence of geometry and causation. In this paper therefore, I will undertake to defend the general drift of that thesis: there are indeed close connections between geometry and causation. The geometry we ascribe to physical space will help determine the nature of our causal laws, and vice-versa. But perhaps I am getting ahead of myself; for reasons which will become apparent later, before I can appropriately deal with this thesis I must first discuss Reichenbach's notion of a universal force.

2. Universal Forces. Universal forces, in contradistinction to differential forces, are ones (i) which affect all substances in exactly the same way and (ii) against which no insulating walls can be built (cf. Reichenbach, p. 13). Reichenbach further distinguishes between universal forces "preserving coincidences" and those "destroying coincidences." The forces preserving coincidences are constant throughout the entire universe and consequently neither the force nor its effects can be objectively determined. The ones destroying coincidences are active only in localized regions of space and therefore are demonstrable. Adolf Grünbaum [Philosophical Problems of Space and Time, 2nd ed. (Boston: Reidel, 1973)] has subsequently characterized those universal forces which preserve coincidences as "metaphorical" and those which destroy coincidences as "literal." It is these "literal" universal forces which will prove most important for our present concerns.

Reichenbach explains that, actually, "The distinction between universal and differential forces merely classifies the phenomenon as belonging in geometry or in physics" (Reichenbach, p. 27). A force is something which is responsible for a geometrical change; consequently the existence of a force is dependent upon the coordinate definition of geometry.

If our measurements indicated that a geometry G' obtained, we could still choose to call G the "actual" geometry and posit a universal force F to explain the discrepancy in our measurements. Thus Reichenbach can

assert, "We obtain a statement about physical reality only if in addition to the geometry G of space its universal field of force F is specified. Only the combination

$$G + F$$

is testable statement" (Reichenbach, p. 33). Simply being told that a geometry G applies will not convey any information about physical space unless we also know what, if any, universal forces are being employed.[2]

3. Causal Anomalies. Suppose we have specified the geometry and the universal field of force (i.e. we have a statement of the form G + F); have we now adequately described the topology of physical space? Reichenbach says no; there still may be causal anomalies which we have not yet considered.

To illustrate, Reichenbach asks us to imagine a system of concentric spherical shells (figure 1) and the experiences of a man climbing about on them (cf. Reichenbach, pp. 63-66). After measuring shell 1, the man proceeds to shell 2. Measuring 2, he finds it to be smaller than 1 and thus concludes that it is situated inside 1. He is astonished however to find 3 as large as 1 and hence larger than 2. In Euclidean geometry, 3 has to be smaller than 2 since it is enclosed by 2. Our explorer chooses to adhere to Euclidean geometry and explains his measurements by a contraction of the measuring rod; his rod and also his body contract so that 3 appears larger than 2. In Reichenbach's terminology, the explorer posits a "universal force" to explain the difference between the "actual" and the "apparent" geometry.[3] Going on to shell 4 he measures it to be even larger than 3; his rod has further contracted. 5 he again finds as large as 3 and 1.

But here his observations startle him. Everything in 5 is familiar to him, including his own house which had been built on shell 1! The correspondence is perfect in every detail; the house is exactly as he had left it. Yet he is quite certain that he is separated from surface 1 by the intervening shells. If he is to retain Euclidean geometry he must conclude that there are two identical worlds; every event on surface 1 is paralleled by an identical event on 5.
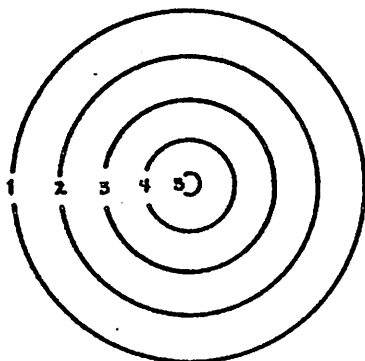
(figure 1):   Concentric Spherical Shells
(to be conceived 3-dimensionally; the circles being
cross sections of spherical shells in the plane of
the drawing)

Figure 1 was drawn to represent the shells on a
Euclidean interpretation. An alternative interpreta-
tion of these bizarre experiences would be to adopt the
geometry of the torus (figure 2). On this interpreta-
tion our explorer starts at the shell whose cross sec-
tion is appropriately labeled "1."   He travels "inward"
toward the "hole" where the cross section of the second
shell is labeled "2," continues traveling around the
torus (his path of travel being represented by the dotted
line) till he is on the "underneath" side at shell 3;
naturally 3 measures larger than 2 and equal to 1.   Shell
4 is the "outer rim" of the torus and 5 is of course
identical with 1, our explorer having come full circle.
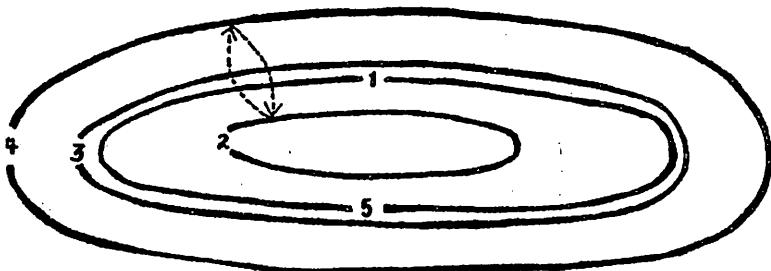
(figure 2):   A Torus

Reichenbach has presented us with a hypothetical
situation in which even positing a universal force

(responsible for the contractions of the measuring rod) would not suffice to allow us to retain Euclidean geometry. In addition, a causal anomaly occurs; there is a spatial periodicity of all events (a kind of pre-established harmony). Thus if our description in Euclidean geometry is to be topologically equivalent to the description in the Non-Euclidean geometry of the torus, we must, in Reichenbach's notation, write $G_0 + F + A$ and not just $G_0 + F$ (where G_0 is Euclidean geometry, F is the universal field of force, and A is the causal anomaly).

The causal anomaly manifests itself because the torus and the plane have different holistic properties (they are topologically different surfaces). While any two surfaces can be mapped one upon the other (as by the projection of shadows by light rays), it is only for surfaces with the same holistic properties that it is possible to carry through this transformation uniquely and continuously in all points. Here I use "uniquely" with Reichenbach to mean: "one and only one point of one surface corresponds to a given point of the other surface, and vice-versa"; i.e. there is a one-to-one function from the points of one surface onto the points of the other. "Continuously means: neighborhood relations in infinitesimal domains are preserved; no tearing of the surface or shifting of relative positions of points occur at any place" (Reichenbach, p. 59). Small regions of topologically different surfaces can be mapped thusly, but no such transformation is possible for the whole surface. It is this violation of continuity and/or uniqueness which corresponds in the physical interpretation to a causal anomaly.

4. The Disappearance of Universal Forces and the Elimination of Causal Anomalies. On the one hand Reichenbach believes that "the question concerning the structure of space" will not be meaningful unless universal forces are set equal to zero; "only then," he says, "is the problem of geometry uniquely determined" (Reichenbach, p. 27; cf. also Carnap's remarks in his "Introductory Remarks to the English Edition" of The Philosophy of Space and Time, p. vii). On the other hand, Reichenbach says that, "the topological character of space . . . is determined only if we add the postulate of the disappearance of causal anomalies" (Reichenbach, p. 279; cf. also p. 80). Propounding both theses at once he says,

> The metric of a space becomes an empirical fact only after the postulate of the disappearance of universal forces is introduced. Similarly, the topology of space becomes an empirical fact only

if we add the postulate of the principle of
action by contact (Reichenbach, p. 279).

Reichenbach would have us set both the universal field
of force F and the causal anomaly A equal to zero.
Unfortunately this is not always possible--consider
another hypothetical situation analogous to Reichenbach's
own:

This time our explorer measures shell 2 to be
smaller than 1, 3 smaller than 2, 4 smaller still, and
5 even smaller. Upon arriving in shell 5 however, his
experiences are even more bizarre than before. Not
only is there a duplication of everything that he left
in shell 1 but, since shell 5 is smaller, everything
else is correspondingly smaller; he is a giant in a
world identical to the one he just left. Upon return-
ing to shell 1, his fiends tell him of a similar giant
who visited in his absence and finally left looking
quite dumbfounded. Wandering off in the other direction
the shells get increasingly larger till, in the fifth
shell, he is a dwarf in a world exactly like the other
two.

If he is to retain Euclidean geometry our explorer
must assume that there is a series of identical worlds
which get progressively smaller as you proceed in one
direction and progressively larger in the other. No
universal forces have been posited, yet a severe causal
anomaly occurs. In other words, we have normalized
F = 0 but cannot, as Reichenbach would have us do,
accept "normal" causality.

If we set A equal to zero however, we will require
$F \neq 0$. If we choose to accept toroid geometry and the
identity of these worlds, we will have to assume the
presence of universal forces which increase the size
of our explorer and his measuring rod when he goes from
shell 2 to 3 and from 3 to 4, but decrease his size
when he goes from 4 to 3 and from 3 to 2. Thus

$$G_0 + A = G + F$$

In this hypothetical situation we can, by the choice
of geometry, set either F or A equal to zero, but not
both. Thus the example squeezes Reichenbach between his
attempt to set F equal to zero and his attempt to have A
equal to zero. He cannot object that such a state of
the universe cannot exist; for we have started from the
perceptions of an observer and, in Reichenbach's own
words, "No a priori postulate can exclude the possibility
that some person may at some time have certain percep-
tions. Only the interpretation of such perceptions is

controversial" (Reichenbach, p. 77). If nothing else,
this casts serious doubt on the universal acceptability
of Reichenbach's twin "methodological" postulates of the
disappearance of universal forces and the elimination of
causal anomalies. If Reichenbach wants to maintain that
we must set F equal to zero, he cannot also require us
to set A equal to zero. If he chooses instead to go
with A having always to equal zero, he cannot require
us to set F = 0. None of this of course precludes him
from insisting either that F must always equal zero or
that A must always equal zero; it does however preclude
him from doing both, and in so doing, casts doubt on
the ultimacy of either.4

"Causal anomaly" is a relative term; an anomaly
under one set of causal laws may be a perfectly normal
causal relation under a different set. For this reason
it may be more felicitous to break with Reichenbach and
speak merely of differing sets of causal laws. The
system we have been calling "normal" causality (with
Reichenbach) is just another system of causality; there
is no reason to believe that it should be uniquely pick-
ed out by stipulating that F = 0. Thus instead of
Reichenbach's claim that we have to accept "normal"
causality for topology to be an empirical matter, we
can say that the system of causality needs only to be
specified; different topologies require different sets
of causal laws.

5. Connections between Geometry and Causation. In
Reichenbach's hypothetical example, as in my own, if
the explorer decides to retain the principle of action
by contact and refuses to admit any kind of pre-
established harmony, then given his experiences he can-
not consistently ascribe Euclidean geometry to physical
space. Conversely, if he chooses to retain Euclidean
geometry at all costs, his experiences will preclude
him from adopting the traditional system of causal laws.
Either alternative would be open to him and his choice
could be fairly arbitrary, but he could not, so to
speak, "have his cake and eat it too": he could not
choose to retain both Euclidean geometry and "normal"
causality.5

None of this shows that the principle of action by
contact is, in fact, incompatible with Euclidean geom-
etry; the question is an empirical one. A given geometry
does not logically entail a specific set of causal laws,
or vice-versa. It is only in conjunction with empirical
experience that beliefs about causality have ramifica-
tions for the geometry of physical space. Similarly,
unless concatinated with experience, geometry does not
tell us anything about causation. But given experiences

such as we have described, adhering to a set of causal
laws will limit our freedom when assigning a geometry
to physical space, and adhering to a geometry will cur-
tail our freedom in formulating "laws" of causality.

Just how thorough-going are these connections be-
tween geometry and causation? Does a set of causal
laws pick out a unique geometry for physical space?
vice-versa? There is of course a rather trivial sense
in which we can answer these questions in the negative:
obviously our experiences can be scant or irrelevant
and there is surely a half-way house where our experi-
ences would suffice to establish some relationships
between geometry and causation, but would simply be
insufficient for a unique prescription. We are thereby
enticed to move on to the more penetrating question as
to whether a system of causal laws, in conjunction with
unlimited experience, will pick out a unique geometry
for physical space, or vice-versa.

This question is not so easy to answer, but we can
begin by recalling that differing sets of causal laws
are required whenever we shift between geometries with
different holistic properties. Any two geometries with
the same holistic properties can be rendered equivalent
by the addition of universal forces. Does this mean
that topologically equivalent geometries will never
require different causal laws? Not necessarily; univer-
sal forces may themselves sometimes constitute a type
of causal anomaly. We must investigate the causal
status of universal forces. To this end it will be
instructive to consider gravitation, although it is not
purely a universal force.[6]

Bertrand Russell, for one, rejoiced in Einstein's
treatment of gravitation. With the adoption of non-
Euclidean geometry, he observed, we no longer need to
assume that the sun somehow "makes" the earth travel in
an ellipse about it; the earth simply travels along the
natural geodesic prescribed by the geometry; no "forces"
or "little pushes" are required to keep it on its orbit.[7]
It is not that we have excused ourselves from giving a
causal explanation of the orbit--we are using geometrical
considerations as our physical explanation, not rejecting
the need for one. The geometry of the universe serves
as our "cause."

In such a case, the change of geometry and the re-
sultant postulation of universal forces entails a cor-
responding change in the causal explanation of certain
phenomena. It may well be that a change in geometry
always entails a change in the explanation of some such
phenomenon. Still, the point of the causal anomaly was

that it represented a change in causal <u>laws</u>; changing our causal <u>explanations</u> of certain phenomena is not the same thing as changing our <u>system</u> of causal <u>laws</u>. We seek to know whether changing from one geometry to another with the same holistic properties involves a change in causal <u>laws</u>.

Part of our difficulties here stem from our ignorance about what actually constitutes a change in causal laws. I suggest that this much is certain: a change in causal laws is involved whenever the explanation under one of the geometries requires a principle not used to explain any phenomena whatsoever under the other geometry. Thus if one geometry could support explanations of all phenomena without invoking action at a distance, but the other geometry could not (as in Reichenbach's hypothetical example), we would say that a change in causal laws was required.

A further source of difficulty may seem to lie in our agreement to allow <u>unlimited</u> experience; how are we to know what unlimited experience would turn up? Fortunately, we are not tied to the experiences of the "real" world; we have recourse to hypothetical situations. Thus we can assume that gravitation is a universal force, pure and simple, and not be bothered about its differential effects (there simply won't be any in our hypothetical worlds).

If we assume that, given enough time, physics will discover some gravitons or demonstrate the existence of the aether, we may conclude that the change in geometry analogous to that proposed by Einstein will not necessitate a change in causal laws; no differentiating principle of causation would be needed. If on the other hand, we assume that even unlimited experience will not turn up such substances, the change in geometry would entail a change in causal laws. Adhering to Euclidean geometry would preclude us from adopting a system of causation which did not sanction action at a distance; the alternative geometry, however, would not require the use of this principle.

So we have one case in which a change between geometries with the same holistic properites requires a change in the system of causal explanation and one case in which it does not. We have already seen that changing between geometries with different holistic properties requires a change in the causal laws. The interesting point about discovering gravitons or aether is that Euclidean geometry would not have to use action-at-a-distance to explain gravitational forces and hence could employ the same set of causal laws as does the Non-Euclidean geometry of Einstein. A change in geometry

does not always necessitate a change in the system of
causal laws.

Now we have left only to investigate whether a
change in causal laws will always entail a change in
geometry; given the same unlimited experiences, is more
than one system of causal laws compatible with any one
geometry? It is important to notice the qualification
"given the same unlimited experiences"; in our contact
under one set of experiences and compatible with action
at a distance under another set. But we did not there-
by show that, given only one set of experiences, we can
make a change in our system of causal laws without
having to change our geometry also.

As one might expect, the real difficulty arises in
trying to find two differing systems of causal laws
which cannot be decided between on the basis of exper-
ience. To this end, the philosophy of Bishop Berkeley
will prove to be a blessing. For Berkeley, to exist is
to be perceived (esse est percipi). Leaving aside
Berkeley's divine perceiver, objects continually pop in
and out of existence; I can make this peiece of paper
cease to exist and then recreate it again simply by
turning my head for a few seconds. Obviously there
must be some sort of causal connection between the ob-
server and the object observed. In fact Berkeley's
system was intended as an alternative to that system
which explains the existence of physical objects by
some kind of underlying substratum or substance (John
Locke's system in particular). Naturally, Berkeley's
system is at variance with present day physics; it
represents a different set of causal laws.

Berkeley's system was never well-accepted; even at
the outset it was thought to be disproved simply by
kicking a stone. But the system is not so easily dis-
proved; to show that it is mistaken one would have to
demonstrate the existence of a physical object at a
time when it was not perceived by anyone.[8] His system
has the distinctive feature that it cannot be rejected
simply by using sensory experience; if we are to reject
the system, it must be on some other grounds.

But the geometry of physical space will not provide
such a basis; there is nothing about causing objects to
exist by perceiving them which requires us to give up
Euclidean, or any other geometry for that matter (if
there were, you can be assured that someone would have
advanced it as an argument against Berkeley). The con-
clusion is obvious: even given the same set of unlimited
experiences there is nothing about changing to the causal
laws dictated by Berkeley's system which would require us
to change the geometry we assign to physical space; a

change in causal laws will not always entail a change in
geometry.

6. Summary. In addition to showing that Reichenbach is
mistaken in wanting to set both F and A equal to zero,
we have established (i) that a change in geometry can
compel a corresponding change in our system of causal
laws,[9] (ii) that a change in geometry can take place
without entailing a change in the system of causal laws,[10]
(iii) that a change in causal laws can require a change
in the geometry we assign to physical space,[11] and (iv)
that a change in causal laws can be made without re-
quiring a change in geometry.[12] Conjoining these four
theses we may say that, though there are times when a
change in causal laws will entail a change in geometry
(and vice-versa), even in conjunction with unlimited
experience, a system of causal laws will not necessarily
suffice to pick out a unique geometry (or vice-versa).
The connections between geometry and causation are
close indeed, but not close enough for a unique pre-
scription.

University of Illinois at Chicago Circle

NOTES

[1]Cf. Hans Reichenbach, The Philosophy of Space and Time, Eng. ed. (New York: Dover, 1958), pp. 80, 279.

[2]It may be helpful to note that the "actual" geometry G', for which F has been set equal to zero, is just that geometry which results when we assume our measuring rods to be rigid.

[3]The force is universal rather than differential since it affects all substances in exactly the same way and cannot be shielded against; it is a literal force which is said to destroy coincidences since it is operative on shell 3, but not on shells 1 and 2.

[4]Incidently, we might still go along with him about setting both equal to zero if possible and use some other criteria (e.g. overall simplicity of the system) to decide which, if either, to set equal to zero if a situation such as we have described should ever arise.

[5]The alternative of disbelieving the sensory experiences is ruled out because the experiences are quite reproducible and the measurements and observations can be repeated as often as one should care to go through the procedure.

[6]Gravitation is universal in that it cannot be shielded against and, in some respects, affects all substances in the exact same way (e.g. the rate of acceleration of a falling body does not vary with the chemical composition of that body). But the force is differential in certain of its other effects (e.g. a wooden bookshelf will sag more under the influence of gravity than a steel one) (cf. Grunbaum pp. 91-92).

[7]Bertrand Russell, An Outline of Philosophy (1927; rpt. New York: World, 1963), pp. 122-23.

[8]Things become further complicated if we introduce the divine perceiver since even this demonstration would fail to disprove the theory; I have left him out for two reasons: (1) unlimited experience should perhaps turn up the existence of such a god, and (2) leaving him out simply makes for a sharper distinction in the systems of causal laws.

[9]This is established for geometries with different holistic properties by Reichenbach's hypothetical example and, for geometries with the same holistic properties, by my reflections on gravitation.

[10]This is shown by my reflections on gravitation for a change from G to G' where G and G' do not have different holistic properties.

[11]This is established by Reichenbach's hypothetical example. Given the experiences of our explorer, if his causal laws preclude pre-established harmony of any sort, he cannot consistently ascribe Euclidean geometry to physical space; which is not to say that his causal laws have uniquely picked out the replacement geometry, though they have presumably ruled out all geometries topologically similar to the Euclidean planar geometry.

[12]This is what we just showed using Berkeley's alternative system.