LEIBNIZ'S MONADIC TREATMENT OF RELATIONS

DONALD MERTZ

I. Introduction

Leibniz understood, perhaps better than most metaphysicans, the importance of relations as the cement that binds possible worlds together, that gives them wholeness and structure, and that makes for their intelligibility.[1] He expresses this, for instance, in his often repeated position that each monadic substance, though self-sufficient and causally isolated from all others, is nevertheless mutually connected with them by an infinity of "relations that express all others," and which make each monad "a perpetual living mirror of the universe."[2] And again he says, "There is no term so absolute or detached as not to include relations, and the perfect analysis of which does not lead to other things and even to all others."[3] It is because of these and other uses of relations in his metaphysics, which contrasted with a brief though interesting logical treatment appearing to eliminate them, that has made Leibniz's theory of relations the subject of increasing philosophical attention. Relatively recent studies in this area have not only served to rectify previous misconceptions, but also to clarify certain aspects of Leibniz's logic and metaphysics.

For example, Russell's classic position, that Leibniz sought to eliminate all relations by a reduction to non-relational attributes, has been shown to be inaccurate.[4] H. Ishiguro[5] and J. Hintikka[6] have argued that Leibniz's often repeated position of the non-existence of relations is not a proposition of his logical theory, but a metaphysical position as to what exist with the same ontic status as monadic substances. They point out that on a close examination of his few

examples, Leibniz's program of a "rational grammer" (Grammaticae Cogitationes)[7] for the re-writing of relational propositions into certain other forms, is not an attempt to eliminate relational concepts, but an attempt to make relational statements amenable to traditional subject/predicate argument forms.[8] Further, Hintikka has shown with the help of modern logic that Leibniz's distinction between "possibility" and "compossibility" would be one without a difference so long as relational concepts are not fully admitted.[9] Likewise, Ishiguro rightly points out that Leibniz's "mirror thesis," that every individual "expresses" or mirrors the whole universe from its point of view, is intelligible only when relations exist among subjects to furnish this intercon-nection.[10]

Continuing, then, in the direction of these studies, it will be the purpose of this paper to explicate further the exact form of Leibniz's brief logical/syntactical program for relations, and then to contrast this with his metaphysical/semantical treatment of them. It will be seen from this analysis that there is a similar though not identical treatment of relations under both programs. This similarity lies in Leibniz's treating multi-term relations as one-term, monadic predicates with all other term-places being either instantiated, or bound by existential quantifiers, depending on the program. It will be evident that Leibniz is not attempting to eliminate relations in favor of attributes, and that in fact his logical program requires his introducing a new relational connective between propositions. Finally, I will show how relations can once again be used to clarify a difficult point of Leibniz's metaphysics. Namely, how it is possible that positive concepts, which go to make up each individual's "complete concept" and so render it distinct, can be either compatible or incompatible. This latter problem puzzled Leibniz and commentators since.

II. The Grammer of Relational Paraphrase

According to Leibniz, relations are of two kinds, those of comparison and those of connection (comparationis vel connexionis).[11] The examples of paraphrase that he gives differ according to whether the relation is one or the other of the two kinds. As examples of relations of comparison he lists those of resemblance, equality, inequality, identity, and diversity, and as examples of relations of connection those of cause and effect, of whole and part, of

position and order, and of predication of existence.[12] It is significant that the former are all symmetric, or more particularly, equivalence relations or their denials, while the latter are asymmetric on non-symmetric relations.

Now, it is clear from the following examples that Leibniz thought that at least some propositions expressing relations of comparison could be reduced to a truth-functional conjunction of propositions of the subject/predicate form, i.e., as containing one-place, monadic predicates. He gives as an example:

> All oblique inferences - e.g., 'Peter is similar to Paul, therefore Paul is similar to Peter' - are to be explained by explanations of words. Such may be seen from the logic of Jungius. It is reduced to the proposition 'Peter is A now' and 'Paul is A now'.[13]

The proposition "Peter is similar to Paul" is here paraphrased into the conjunction of the two attribute-like propositions, "Peter is A now" and "Paul is A now." This is a common sense reduction, for to say that two subjects are similar is to say that they share some common property which as such renders them alike. But more precisely, the form of the above paraphrase implies that the predicate "is similar to" is something like a second order relation that exists among the two subjects, Peter and Paul, and this due to the fact that both subjects possess the same first order predicate, "is A now." Though Leibniz's paraphrase eliminates as explicit the second order relation "is similar to," it retains the shared first order predicate, and importantly, it is possible for this shared predicate to be either an attribute or a relation. For, even though "is A now" is syntactically monadic, A could be intensionally either a true monadic attribute, e.g., "is just," or yet remain a relation, e.g., in the form of "is taller than John." In the latter case, to use modern parlence, we have a two-place relational predicate with only one free varibale, 'x is taller than John,' where the variable x would be instantiated by both Peter and Paul. In general, since the above reduction is possible for any equivalence relation, it would cover all cases of Leibniz's relations of comparison.

In examples dealing with the asymmetric or non-symmetric relations of connection, Leibniz is more explicit in retaining relational predicates. He gives as an example the sentence "Titus is wiser that Caius"

which, using a special logical connective, he re-writes as,

> Titus is wise and as such (qua talis) is superior, to the extent that (quatenus), Caius qua wise is inferior.[14]

Here there is no elimination of relational predicates because the resulting two predicates of the paraphrase are respectively the relations "is superior in wisdom" and "is inferior in wisdom." These predicates are binary relations made implicit by the suppression of one relata. That is, Titus can be superior in wisdom only with respect to another subject who is not as wise, whether this latter subject is given explicitly or not. In leaving the second relata unspecified in both predicates, Leibniz is in effect treating these two-place relations as syntactically monadic. This procedure of regarding relations as one-place predicates without specifying the all but one suppressed relata is common to modern logic and is handled with the use of bound variables. For example, the sentence "Titus is wise and as such is superior" would translate to "(x)(Titus is wiser than x)." Likewise, "Caius qua wise is inferior" becomes "(y)(y is wiser than Caius)." Written as such, the predicate (x)(___ is wiser than x) is seen as a relational though one-place predicate attributable to Titus.

Thus, Leibniz would appear so far to be following a simple grammatical procedure; that for relations to be treated grammatically like attributes, they must, like attributes, be predicated ostensively of one subject. This requires that the other relata be suppressed, or, as we would now say, referred to indefinitely by the use of an existential quantifier which ranges over variables instantiated by the suppressed relata. This procedure, however, has the disadvantage of requiring Leibniz to introduce special propositional connectives to circuitously re-establish the connection between the given relata, a connection that is lost when the relata are suppressed in the paraphrase.

An understanding of Leibniz's need for the special propositional connective "to the extent that," or alternately, "by virtue of the fact," comes upon observing that the simple conjoining of the two paraphrased propositions by the ordinary truth-functional connectives is inadequate to determine an equivalence to the original proposition. For example, the conjunction, (x)(y)(Titus is wiser than x, and y is wiser than Caius), is too weak in that it does not

imply the original proposition, Titus is wiser than Caius. This is also true when a biconditional is used. The failure of an equivalence with the use of truth-functional connectives is due to the fact that when they are used the two specific relata, Titus and Caius, remain unrelated. All we know, for example, by the use of a conjunction "and," as above, is that there is someone who Titus is wiser than, and that there is someone, not necessarily the same person, who is wiser than Caius.

More than a standard truth-functional connective is needed, therefore, to link the internal propositional constituents of Titus and Caius as the actual two subjects being related here. Leibniz is aware of this in introducing the special connective "to the extent that." What he intended by this connective is unclear, however I would offer the following as a plausible interpretation. Leibniz's paraphrase could be re-written as: By the same instance of the relation "is wiser than," Titus is superior in wisdom and Caius is inferior in wisdom. This accomplishes the task of re-establishing the link between Titus and Caius without mentioning them, and does so in the only other possible way by referring to the relation they share in the particular instance in which they share it. This interpretation also has the necessary advantage of being logically equivalent to the original proposition, Titus is wiser that Caius. That this is Leibniz's intent may come out more clearly in the following example.

Leibniz writes,

> This will be the best way of explaining 'Paris is the lover of Helen', that is, 'Paris loves, and by that very fact (et eo ipso) Helen is loved.' Here, therefore, two propositions have been brought together and abbreviated into one. Or, 'Paris is a lover, and by that very fact Helen is a loved one.'[15]

Here again the two resulting sentences contain concealed dyadic predicates which are represented as syntactically monadic, and again with one bound variable. Partially translated, Paris loves someone, and Helen is loved by someone, i.e., (x)(Paris loves x) and (y)(y loves Helen). Here, as in the previous example, the special connective, et eo ipso, shows that the original proposition is not simply a truth-functional conjunction of the two paraphrase constituents. The conjunction "Paris loves and Helen

is loved" does not have the meaning, nor is truth-functional equivalent to, the original proposition expressed by "Paris is the lover of Helen." This is so here, as in the previous example, because there is no link, stated or that can be inferred, connecting to each other the two relata, Paris and Helen. Since this connection can only be achieved by referring to the same instance of the relation of loving, the connective et eo ipso must translate the same as the previous quatenus. That is, "and by that very fact" must translate to: By the same instance of the relation of loving, Paris loves y, and x loves Helen. The special connective will then serve to identify exactly who is loving who.

From the above examples it is evident that Leibniz is not attempting to eliminate relational predicates by a reduction to true monadic attributes. Relations, in fact, remain elements of the paraphrases, but are treated as syntactically monadic, and here in the particular mode of containing one bound variable. Moreover, it is this bound variable mode that requires Leibniz to introduce these connectives. As will become clear below, his treatment of relations as monadic predicates containing instantiated rather than bound variables requires no such analog to these connectives. In addition, it is important to note that the special connectives, which under our interpretation turn out to be the same, are themselves relations between propositions. They are, as it were, second order relations which serve to combine the two monadic, attribute-like occurrences of the re-written first order relations. At any rate, from the vantage of modern logic Leibniz's grammatical program is awkward, to say the least. It is, nevertheless, a clear attempt to retain relations but to make them function syntactically as one place predicates. As will be seen, this program is analogous to his metaphysical/semantical treatment of relations, though with a significant difference.

III. Relations in Complete Concepts

In contrast to the above, a formal explication of Leibniz's metaphysical/semantical program for relations is more immediate. Semantics is concerned with the interpretation of signs whereby they have meaning and/or truth, and for Leibniz the fundamental unit of truth is the proposition. However, propositions are complex terms whose constituent simple terms stand for the fundamental units of meaning, i.e., concepts of

ideas.[16] What Leibniz understood by a concept or idea is very similar to the Platonic theory of Forms, as he himself points out. An idea is an immediate object of thought, and is a permanent form or essence, ultimately existing and objectified in the mind of God.[17]

Leibniz's constant and repeated position is that an affirmative proposition is true when the predicate concept is a constituent of the subject concept, or is identical with it.[18] This is no less the case for relational propositions, i.e., those of "extrinsic denomination."

> . . . A reason can be given for every truth (which is not identical or immediate), that is, that the notion of the predicate is always expressely or implicitly contained in the notion of its subject, and this holds good no less in extrinsic than an intrinsic denominations, no less in contingent than in necessary truths . . .[19]

Further, when the subject of a proposition is an individual substance, or monad, the predicate is contained in what Leibniz calls the "complete concept" of that individual. By a complete concept he means a maximal set of compatible simple predicates satisfiable by exactly one individual. This set is maximal in the sense that the addition of any proposed concept would either be redundant or make for a contradiction. A "possible world" is then, in turn, a maximal set of these complete concepts which are mutually compossible.[20] In any possible world each individual's complete concept is a plenum of <u>all</u> mutually compatible attributes and relations, past, present, and future that the corresponding substance will ever have in that world. In particular, this abundance includes infinitely many relations connecting each individual to every other.

> Now this mutual connection or accomodation of all created things to each other and of each to all the rest causes each simple substance to have relations which express all the others and consequently to be a perpetual living mirror of the universe.[21]

Leibniz states that by a monad expressing the universe he means that there is "a constant and regulated relation between what can be said (i.e., the predicates) of the one and of the other . . ."[22] That is, the constituent concepts of any given individual's

complete concept must be reflected in some manner in the constituent concepts of every other complete concept.

The pertinent point here is the specific manner in which relations are contained in and are reflected amongst these individual concepts. Leibniz writes,

> I say that the concept of an individual substance includes all its events and all its denominations, even those which are commonly called extrinsic, that is, those which pertain to it only by virtue of the general connection of the things and from the fact that it expresses the whole universe in its own way.[23]

Elsewhere he writes, " . . . there must be a plurality of affections and relations in the simple substance, even though it has no parts."[24] In other words, an analysis of an individual's complete concept by a sufficient intelligence would ultimately lead to the knowledge of every substance, each as a relata in its mirrored correspondence with that individual. As we saw earlier, "There is no term so absolute or detached as not to include relations, and the perfect analysis of which does not lead to other things and even to all others."[25] If we state this formally, Leibniz's position is equivalent to the fact that relational predicates are contained in their subjects as multi-term predicates, but with all the terms given or instantiated, save one. For example, an analysis of the complete concept of Paris would yield not the two-place relational predicate "x loves y," but the one-place relational predicate "x loves Helen." In general, an n-place relational predicate would be contained in its subject as a predicate with n-1 instantiated variables, and one free variable in the place of the subject from which it was analyzed.

It is, then, this monadic mode of relational containment that provides the mechanism for explaining how an analysis of a singular concept, such as that of Paris, can lead to, and thus "express" from its own "perspective," another singular concept, e.g., that of Helen. As such, this special containment functions as the metaphysical analog of the special connectives of Leibniz's grammatical program. In both treatments, relations are not eliminated in favor of simple attributes, but are retained in the attribute-like form of monadic predicates. The specific form differs according to Leibniz's purpose; suppressed relata in

the form of bound variables in the one, fully given and instantiated relata in the other.

IV. The Necessity of Relations

Rendering explicit the monadic containment of relations within complex concepts helps to clarify certain of Leibniz's most ambiguous and misinterpreted passages and positions. To take just one example of such a passage, Leibniz refers to the proposition: "David is the father of Solomon."

In reference to this proposition he maintains that:

> This I hold, as regards relations, that paternity in David is one thing and filiation in Solomon another, but the relation common to both is a mere mental thing, of which the modifications of singulars are the foundation.[26]

The "modifications of singulars" here would be the monadic relational predicates "x is the father of Solomon" (i.e., filiation in Solomon) and "David is the father of y" (i.e., paternity in David). The "mere mental thing" is the relation "common to both," in other words, the abstract relation "x is father of y," which as such is totally uninstantiated and unconnected to any individual. Likewise, Leibniz states in another misinterpreted passage, "Therefore we must say that this relation in this third way of considering it (i.e., x is greater thay y) is indeed out of the subject; Being neither a substance, nor an accident it must be a mere ideal thing . . ."[27] The point is that relations have their "foundation in things"[28] precisely in their monadic mode, but when otherwise abstracted from any relata they are merely ens mentale.[29] Therefore, this passage can not be interpreted as implying that relations are trivial and can in the last analysis be eliminated, nor that they are necessary only in some psychological sense. Relations in the monadic mode have the same ontic status as any of the other concepts that they do combine with, and concepts are for Leibniz "absolute forms."[30]

Finally, it is interesting to see yet another case where relations, particularly in their monadic/instantiated form, can be used to clarify an otherwise difficult point in Leibniz's metaphysics. As indicated above, an individual's complete concept is a class composed of an infinity of simple concepts.

Leibniz states that all simple attributes are 'positive,' and that ". . . all purely positive terms are compatible inter se."[32] The question then becomes; how is it possible for any class of positive concepts to be compatible or incompatible with any other such class? That is, how is it that one object's having a positive predicate P can logically imply or exclude another object's having a positive predicate Q. There must be some incompatible simple concepts for otherwise there would be but one complete concept and therefore but one individual. Leibniz, as commentators after, found this to be a great mystery understandable by God but not by man.[33]

This problem is easily solved, however, when one extends "simple attributes" to include relational predicates in the above monadic form, i.e., instantiated with all relata save one for the subject. For example, if we let P be the simple and positive predicate "x loves Helen," and likewise Q to be "Paris loves y," then the individual Paris in having the predicate P logically implies that the individual Helen has the predicate Q, and conversely. Therefore, the predicates P and Q are compatible, and to that extent so are the containing concepts of Paris and Helen. Or, take as an example the case where individual A is taller than individual B. The complete concept of A, insofar as it contains the positive monadic predicate "x is taller than B," implies and is therefore compatible with the complete concept of B, insofar as it contains the predicate "A is taller than y." On the other hand, if A had a complete concept that included the predicate "x is taller than B" and B a complete concept that included the predicate "x is taller than A," then A and B would be logically incompatible in both being taller than the other. In such a case Leibniz is certainly correct in saying that A and B could not both occupy the same possible world. In brief, it is possible to make a distinction between compatible and incompatible positive relational concepts, and this distinction depends upon both the position of the specific relata in the n-place relation, as well as the kind of relation involved, i.e., reflexive/irreflexive, symmetric/assymmetric, transitive/intransitive. Recall that it was by means of the asymmetric property of some relations that Russell argued against Leibniz's supposed position, demonstrating that such relations are not reducible to attributes.[34] Therefore, in saying that all positive terms are compatible inter se, Leibniz's difficulty arises in restricting "terms" to include only simple attributes, to the exclusion of monadic relational

predicates. For, just as Hintikka demonstrated the necessity of relations to found the distinction between possibility and compossibility, so we see here their role in rendering intelligible the difference between compatible and incompatible positive concepts.

Saint Louis University

NOTES

[1]In regard to the latter one need only think of the first two hypotheses of Plato's Parmenides.

[2]Leroy Loemker, Gottfried Wilhelm Leibniz: Philosophical Papers and Letters (Chicago: University of Chicago Press, 1956), pp. 1053-54.

[3]C. I. Gerhardt, Die Philosophische Schriften von Gottfried Wilhelm Leibniz, 7 vols. (Berlin, 1875-90), vol. V, p. 211. Hereafter abreviated as G. Translation from G. Leibniz, New Essays Concerning Human Understanding, trans. A. Langley (Chicago: Open Court, 1916), p. 236. Leibniz states that relations form the largest field of our knowledge, and that indeed the "reality of truths" consists in the known relation between the subject and predicate of a true proposition. G.II.438. See Loemker, pp. 282-83, 493.

[4]Bertrand Russell, A Critical Exposition of the Philosophy of Leibniz (Cambridge: Cambridge University Press, 1900). This reductionist thesis is also ascribed to by Nicholas Rescher, The Philosophy of Leibniz (Englewood Cliffs, New Jersey: Prentice Hall, 1967), and G. H. R. Parkinson, Logic and Reality in Leibniz's Metaphysics (Oxford: Clarendon Press, 1965). This appears also to be the position of Benson Mates, "Leibniz on Possible Worlds," in Leibniz: A Collection of Critical Essays, ed. H. Frankfurt (Notre Dame: University of Notre Dame Press, 1972), pp. 335-64.

[5]Hide Ishiguro, Leibniz's Philosophy of Logic and Language (Ithaca: Cornell University Press, 1972). Also see Ishiguro, 'Leibniz's Theory of the Ideality of Relations,' in Leibniz: A Collection of Critical Essays., pp. 191-213.

[6]Jaakko Hintikka, "Leibniz on Plenitude, Relations, and the 'Reign of Law'," in Leibniz: A Collection of Critical Essays, op. cit., pp. 155-90.

[7]Louis Couturat, Opuscules et fragments inedits de Leibniz (Paris, 1903), pp. 286-87.

[8]Ishiguro, Logic and Language, pp. 90-93, and Hintikka, pp. 160-68. Rescher nevertheless holds to the reductionist thesis even though arguing that Leibniz's thesis of the non-existence of relations is not a logical thesis but an ontological one. op. cit., pp. 77-78. Leibniz greatly admired the work of the German logician Joachim Jungius (1587-1657) who in his Logica Hamburgensis (1638) attempted an enlargement of traditional logic to account for relational arguments. Leibniz chose a grammatical approach so as to "escape without being compelled, with Jungius, to invent new ways of reasoning . . ." Couturat, p. 289. See E. Ashworth, "Joachim Jungius and the Logic of Relations," Archiv fur Geschichte der Philosophie, vol. 49, 1967, pp. 72-85.

[9]Hintikka, pp. 158-61.

[10]Ishiguro, Ideality of Relations, op. cit., note 5, pp. 197, 213.

[11]Couturat, p. 355.

[12]New Essays, pp. 144, 401.

[13]Couturat, p. 244.

[14]Couturat, p. 280.

[15]Couturat, p. 287.

[16]Couturat, pp. 512, 382, and 243.

[17]Philip Wiener, Leibniz Selections (New York: Charles Scribner's Sons, 1951), pp. 327-30.

[18]See Loemker, pp. 362-63, 367, 472, 517, and 987.

[19]Wiener, p. 94; G.VII.199

[20]See Mates, op. cit., note 4.

[21]Loemker, pp. 1053-54.

[22]Couturat, p. 15; G.II.112.

[23]Loemker, pp. 517; G.II.56.

[24]Loemker, p. 1045.

[25]Op. Cit., note 3.

[26]G.II.486.

[27]Loemker, pp. 1147-48.

[28]G.V.210.

[29]G.II.486.

[30]Loemker, p. 254.

[31]Couturat, p. 35.

[32]G.VII.195.

[33]See Mates, op. cit., pp. 338-40.

[34]Russell, op. cit.,p. 12ff. Also Bertrand Russell, The Principles of Mathematics (New York: W. W. Norton, 1903), pp. 221-24. Before Russell, C. S. Peirce had argued against such a reduction to attributes. See my "Peirce: Logic, Categories, and Triads," Transactions of the Charles S. Peirce Society, XV, (1979), No. 2, pp. 158-75.