A CHARACTERIZATION of MATHEMATICS

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It is alleged by some that there is a crisis in the foundations of mathematics. There are too many propositions in various branches of mathematics which have been shown to be independent of the more widely accepted axioms of set theory. What is needed is for mathematicians to get together and decide once and for all (or at least until the next crisis) which axioms are to comprise set theory and, by doing so, decide what is to be the subject matter of mathematics, i.e., the theory of the chosen axioms. In jumping one way or the other on the axiom of constructibility, the axiom of choice, the continuum hypothesis, Martin's axiom, etc., the propositions referred to above would no longer be independent. A decision on these axioms would determine these statements or their negations as derivable from set theory, and all would be well again (for the time being) with mathematics. Of course, the first response is 'OK, what axioms do you want?' and thereupon begins the whole dispute again--one mathematician wants the continuum hypothesis, another doesn't, and so on.

I think, however, that this allegation and the proposed solution betray a false understanding of mathematics as a monolithic structure built on a single foundation of logic and set theory. Mathematics is said then to consist formally of those theorems derivable from such a single foundation. Or the subject matter of mathematics is said to be fundamentally the properties of sets as uniquely described by set theory. The vista of many crystal palaces, each one built on its own peculiar foundation, is viewed with distaste, especially when some of these towers are unable to decide whether various structures can be incorporated.

The purpose of set theory, it is said, is to provide a single, solid foundation for the whole of mathematics by giving as precise as possible a characterization of the notion of set. It should, as it were, determine as far as possible the architectural

possibilities of the crystal palace of mathematics. No reasonable proposition should be left loitering on the doorstep; it should be admitted or sent away. And if the doorkeeper set theory can't decide, he should be given the criteria to decide: another axiom. But if this is what is required of set theory, the inevitability would be many crystal palaces anyway, each fawned over by its own admirers. That is, mathematics would still be fragmented and the only thing new would be the dispute among mathematicians as to who really are the mathematicians.

I think this is a false view of mathematics. The actual landscape of practicing mathematicians seems to be a veritable suburban development of crystal palaces. As practiced, mathematics consists not of a monolithic structure built upon a single logic and set theory, but rather upon many set theories and, for that matter, many logics. The attitude I would engender is the acceptance of what is the present practice and to see it all as mathematics proper. I would like to characterize mathematics so that the din of the crisis-mongers is reduced, so that the different versions of set theory and logic are all seen as part of the same subject matter--mathematics. I think that if mathematics is properly characterized, then the alleged crisis would be seen as no crisis at all; there would be no more crisis in set theory than there is in, say, group theory.

The following sketchy characterization seems to me to be obvious. Mathematics consists of the description of kinds of things by way of basic propositions called axioms or postulates and of the subsequent deduction of derived properties of these kinds defined by the axioms. Let me move right to an example, for I think the prima facie obviousness of this view arises out of a consideration of many examples of the practices of mathematicians doing mathematics. A group theorist says that a group is set with a two-place operation defined on the set such that the operation is associative, that the group has an identity with respect to the operation, and that each element of the group has an inverse with respect to that operation. Having done this, the group theorist proceeds to give examples of groups: the integers with addition, the rationals with multiplication, etc. He then shows certain derived properties of groups (e.g., the uniqueness of identities and inverses). He defines properties of groups (finite, cyclic, abelian) and shows, by way of example, that some groups have these properties and some do not. He defines other kinds of things that have to do with groups such as subgroups, homomorphism,

and centers; and he discusses properties and kinds of these.

I suggest that the mathematics of group theory is precisely as it is described above as the practice of a group theorist. I don't mean to suggest, by the way, that mathematics is just whatever practicing mathematicians do--rather, I take this as my starting point in my attempt to characterize mathematics in a more enlightening way. What a group theorist does is to define a kind of thing, a group. This is done by presenting the axioms of group theory. The derived properties of groups are shown to be properties of groups by giving a deduction of them from the axioms. If we think of group theory in the way a logician might (that is, rendered in the language of first-order predicate logic with identity), then showing examples of groups amounts to showing that models of group theory exist. In showing that some groups are cyclic and others are not, he has distinguished then two kinds of groups, in fact, two species of the genus group, where the differentia are expressed by the definitions of cyclic and acyclic. In the logician's view, he has shown that if another axiom is added to the axioms of group theory, then one gets another theory (the theory of cyclic groups, for example) about another kind of object which is a species of group.

I think that, in general, the same sort of characterization applies to all branches of mathematics. Group theorists, ring theorists, topologists, measure theorists, and geometers are all engaged in laying down a definition of a kind of thing, deriving properties of it, demarcating species of it, defining related kinds of things and deriving the properties and relations of these. And I think it is clear that these other particular cases can be handled in the same way that I have characterized group theory.

But what has this to do with the foundations of mathematics and set theory? After all, all these areas of mathematics can be done within set theory. For example, a group is an ordered pair with a set as first coordinate and an operation as second coordinate. And an operation is just a set of ordered triples with certain properties specifiable in set theory. Thus, group theory is merely a part of set theory. One can admit the above characterization of mathematics and still be concerned with set theory and the foundations of mathematics. And this concern can spread to group theory since the question of whether certain groups have certain properties or whether certain kinds of groups exist depends on which set theory one chooses.

The set-theoretical rendering of group theory does indeed give the appearance that group theory is merely

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part of set theory. And this gives the appearance that set theory is more fundamental than group theory and any other mathematical theory that can be developed in set theory. Whether this is the correct view of the matter (and I will argue later that it isn't), I think nonetheless that the group theory example is instructive since the characterization given for it might encounter less resistance. For example, a Platonist might agree with my characterization while still being puzzled about which set theory is the correct one. Yet I would characterize set theory in more or less the same way as I have group theory. There is a genus, universe ˙of sets, defined by the collection of, say, the 'noncontroversial' axioms of set theory (if there be such). The set theorist derives properties of sets from the given axioms, defines certain kinds of sets, properties, relations, etc., and proves various propositions about them.

Now under this genus, universe of sets, there are many species of universes, each one differentiated by the various 'controversial' axioms. In other words, all the various set theories merely delineate different species of one genus. For one species of universe the axiom of choice is true and for another it is false, and similarly for the other axioms. The question of whether the axiom of choice is true in some less qualified sense is, then, just like the question in group theory whether the abelian 'axiom' is true (i.e., whether all groups are abelian). The question then appears to be phrased incorrectly. The point is not whether the abelian 'axiom' is true, rather, some kinds of objects which satisfy the group postulates also satisfy the abelian 'axiom' and some do not. If one adds the abelian 'axiom' to group theory one gets the theory of abelian groups. And so it is in set theory, except for a lack of vocabulary. The point is not whether the axiom of choice is true, rather, some kinds of objects which satisfy the other axioms of set theory also satisfy the axiom of choice and some do not. And yet, group theorists lose no sleep over the abelian 'axiom'. There are many propositions which are independent of and consistent with the axioms of group theory. The existence of both abelian and nonabelian groups shows the independence and consistency of the abelian 'axiom'. But there is no crisis alleged in the foundations of group theory; and I claim that there is, in just the same way, no crisis in the foundations of set theory or mathematics.

As I mentioned, Platonists might agree with my characterization of group theory yet balk at extending it to set theory. The thought is that there are eternal, mathematical objects 'out there' whose proper-

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ties one studies in mathematics. For example, among those objects there is the set of real numbers and it is thought that the continuum hypothesis is true of this set or it is not. And unfortunately, the other axioms of set theory do not decide this question one way or the other. Hence, a crisis in the foundations of mathematics is announced; or the Platonist claims that the problem is that we have an insufficiently clear intuition of the mathematical object we are trying to describe and thus are unable to incorporate all of its features into our axiom system.

But why must we think that there is the set of real numbers of which the continuum hypothesis is true or false? Rather, I would suggest that there are various kinds of sets of all real numbers, some of which have the size the continuum hypothesis asserts that they have and some of which are larger. The question is not whether the continuum hypothesis is true of the real numbers, but which set of all real numbers one is talking about in a particular case. And this view is compatible with Platonism--among the eternal objects 'out there' are many sets which are species of the genus set of all real numbers. In the same way that it is not quite to the point to ask whether the abelian 'axiom' is true of groups, even for the Platonist, so it is not quite to the point to ask whether the continuum hypothesis is true of the real numbers. If the Platonist can countenance species of groups, then he can also accept species of real numbers and, for that matter, species of universes of sets.

It can also be seen now that we don't have to retreat to a kind of formalism in which the truth value of the continuum hypothesis is of no concern since no mathematical statements have a truth value, nor do we have to deny the law of excluded middle and say that the continuum hypothesis is neither true nor false. In fact, if the continuum hypothesis asserts that any version of the real numbers has cardinality $\aleph_1$, then it is just false. There are some versions of the set of real numbers which assert that the cardinality of the real numbers is larger than $\aleph_1$.

Thus, there is really no crisis in the foundations of mathematics. The independence and relative consistency of the continuum hypothesis and the axiom of choice and the others do not create a crisis, rather they are interesting mathematical facts. The independence and consistency of propositions in various branches of mathematics do not indicate that there are loose ends in the characterization of the subject matter of mathematics. They do indicate that of the things defined in these branches of mathematics, some have the property in question and some do not.

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Moreover, the proofs of independence and consistency of these propositions are themselves bona fide parts of mathematical knowledge and inquiry. To insist that set theory be so constituted as to decide these propositions would be to deny a place in the subject matter of mathematics to something which very plausibly is a part of the subject matter of mathematics even if one construes mathematics to be only the theory of sets.

On the view that the various set theories are species of universe of sets, independence and consistency proofs show places where generalizations might be sought. What I mean is this: it is usually suggested that in proofs which use, say, the axiom of choice, such a use should be noted. Also, where propositions are proved using the axiom of choice, it is of some interest to find alternate proofs of the same proposition which do not use this axiom. On my view, what is achieved here, when such a proof is found, is greater generality. The reason for trying to prove propositions without using the axiom of choice is not because we don't know whether it is true or false, rather it is in order to have more generality. Propositions proved without the axiom of choice are true both for universes in which the axiom is true and for those in which it is false. It is the same situation as in group theory in which a proof of a proposition that all abelian groups have a certain property is generalized to show that all groups, abelian and nonabelian, have this property.

Some of the reasons for wanting a single set theory arise out of the logicist view of mathematics, that mathematics is part of logic in the sense that all the concepts of mathematics (or at least those which are developed out of number theory) can be defined by way of strictly logical concepts and that the theorems of mathematics can be proved from strictly logical principles using only rules of inference from logic. Whether or not this can be shown, there has been progress in showing that all or most of mathematics can be done on the basis of set theory axioms. Hence, the view arises that whether or not set theory is part of logic, at least mathematics can be reduced to set theory and thus, the subject matter of mathematics (or most of it) is just the notion of a universe of sets as described by the axioms of set theory. The question of which axioms are to be the axioms of set theory then is the question of what is the single, unified subject matter of mathematics.

Now one might admit that there is no point to insisting on a single set theory, that there are many species of universe of sets, but still claim that the result is various species of subject matter of

mathematics in the sense that each species consists of
just what can be shown to hold of the particular
universe of sets determined by the set-theoretic axioms
which are used. It is still claimed that the other
branches of mathematics are just developments of
whatever species of set theory is being used.   Thus,
group theory, topology, measure theory,number theory,
etc., are all seen as the study of certain kinds of
sets whose properties are determined to whatever extent
is possible by the set theory being used. I claim that
this is still a mistaken view of mathematics, that
claims to have reduced mathematics to set theory, or
even to a collection of species of set theories, do not
give a correct account of the subject matter of
mathematics.

What I want to focus on here are the claims that
various sorts of mathematical objects can be 'reduced'
to other, perhaps more fundamental, mathematical ob-
jects. In general, the reductions end up with charac-
terizations of various mathematical objects as sets.
For example, the study of natural numbers is seen to be
the study of certain sets of sets. The natural number
2, for example, is represented by the set of all sets x
which satisfy the following property: there is an ob-
ject b and an object c such that bϵx and cϵx, and for
any z, if zϵx, then z=b or z=c. The other natural num-
bers are represented similarly. Zero is represented by
the empty set and the successor operation is charac-
terized in the usual way. Under this characterization,
one can· show that the Peano axioms for number theory
are provable from the axioms of set theory.   But does
this show that number theory is really only a part of
set theory? Are the natural numbers strictly identical
to the sets of sets characterized above?

What it does show is that there are objects in the
domain of set theory which have some of the same struc-
ture as the natural numbers. The additional claim that
the sets of sets characterized above <u>are</u> the natural
numbers, though, seems to me rather dubious. After
all, there are other set-theoretic ways of charac-
terizing the natural numbers. For example, the stan-
dard way is to represent zero as the empty set and the
successor operation such that if x is a set, then the
successor of x is the set x$\cup${x}. A set which contains
the empty set and which contains the successor of each
of its elements is called inductive.   The set of
natural numbers is then defined to be the intersection
of all inductive sets. In particular, 1 is the succes-
sor of 0 (i.e., 1={ϕ}), 2 is 1$\cup${1}={ϕ,{ϕ}}, etc. With this
characterization, again the Peano axioms can be shown
to be theorems of set theory. This method differs from
the first essentially by picking a particular set of

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the appropriate number of elements as a sort of representative. What is common to the two methods is that the sets used to represent the natural numbers do so in virtue of having the right number of elements.

But even having the right number of elements is not required for a set to represent the natural numbers. For we can represent zero with the empty set and represent the successor operation such that the successor of a set x is {x}. In this case, the natural numbers are represented by the smallest inductive set under this successor operation. On this representation, the number 3, for example, is represented by {{{ ϕ }}}. This characterization also gives the same formal properties to the set of natural numbers as the others do, in the sense that the Peano axioms are provably true of this set. Yet each natural number, on this characterization, is a set of only one element (except for zero: but we could have started with something other than the empty set for zero). What makes this characterization work is not the number of elements in each set, but the number of pairs of brackets used in constructing each set.

Finally, in addition to the plethora of set-theoretic characterizations of the natural numbers, there are non-set-theoretic characterizations. There is a geometric characterization as the collection of evenly spaced points on a half-line starting with the end-point as zero. The successor of a point x is the point closest to x which is not between 0 and x. The point of these examples is to raise the question of why any of them should be identified with the natural numbers. Whatever the natural numbers really are, it is clear that they have a formal similarity to all of the above characterizations, a formal similarity captured in the axioms of number theory. But whatever the natural numbers really are, it is also clear that they cannot be identical to each of the above characterizations. What is not clear is why it should be the case that a natural number is a set of any kind.

It is, however, plausible that number is a property of sets. And, in fact, this is what makes the above characterizations work: with respect to number, the various characterizations give certain sets the appropriate properties so that they have in common the formal properties of the natural numbers. The first two characterizations associate natural numbers with sets or sets with the appropriate number of elements, the third with sets which have the right number of brackets, and the geometric characterization with points which are the appropriate number of units distant from the endpoint of a half-line. What I think this illustrates is not that any one of these charac-

terizations is actually a definition of the natural numbers, rather that there are mathematical structures in various branches of mathematics, parts of which have the same properties with respect to number. In line with my characterization of mathematics, the Peano axioms define a kind of mathematical structure of which set theory and geometry can provide species or instances. What this means is that these structures share number-theoretic properties, not that any one of them literally is the set of natural numbers.

I think similar results apply to efforts to 'reduce' any mathematical structure to another mathematical structure. There is a standard set-theoretic definition of ordered pair $(<a,b>=\{\{a\},\{a,b\}\})$, but there are many other ways of achieving the same result (e.g., $<a,b>=\{\{b\},\{a,b\}\}$ or $<a,b>=\{\{a\},\{b,\phi\}\}$). What is important is not that any of these be identified with the notion of ordered pair, rather it is that all these characterizations have the appropriate character of 'ordered-pairness,' that is, a structure of two objects which are related antisymmetrically. For similar reasons I would urge that group theory is, after all, not merely a part of set theory, though there are objects describable in set theory which have group-theoretic character.

The result is that despite the apparent unification of much of mathematics into set theory, mathematics is best described as a rather decentralized area of inquiry in which the various branches of mathematics are in fact mathematical in their own right and not because they can be 'reduced' to set theory. Each area describes its own object of study with rigor and establishes the derived properties of these objects. Thus, mathematics is not merely a collection of crystal palaces built on various set theories; in addition there are independent crystal palaces for number theory, group theory, and the rest. The various reductions have shown not that mathematics is set theory, but that there are objects describable in set theory which are formally similar to the objects described in other branches of mathematics.

THE DECORATION-ILLUSTRATION-ART TRICHOTOMY

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In the conflict between referentialist and non-referentialist theories of art, perhaps the clearest extremes are the views of Clive Bell and Leo Tolstoy. Tolstoy makes the statement, "If only the spectators or auditors are infected by the feeling which the author has felt, it is art."[1] On the contrary, Bell, the twentieth-century champion of art-for-art's-sake, says, "The representative element in a work of art may or may not be harmful; always it is irrelevant. For to appreciate a work of art we need bring with us nothing from life, no knowledge of its ideas and affairs, no familiarity with its emotions."[2] For Bell, the usefulness of art for emotional purposes is as irrelevant as its usefulness for any other pragmatic purpose. The only emotion relevant to the experience of art, he says, is the "purely aesthetic appreciation" of a "significant form".[3]

If referentialist theories tend to err on the side of confusing artistic merit with successful illustration (in Tolstoy's case, the illustration of emotions), non-referentialists are equally prone to confuse art with mere decoration. The purpose of this paper is to establish a ground for distinguishing artistic worth from both decoration and illustration. This distinction will also show that the relationship between the work of art and the spectator's emotions is different from any of the ones generally presupposed by both referentialists and non-referentialists. The possibility ignored by both camps is that a human being may have emotions that take the abstract rather than the concrete as their object, and that genuinely worthy art will cater only to those emotions pertaining to abstractions--such as, for example, love in the abstract, or hypothetical suffering, or the universal aspects of the human condition (including whatever general structures of human motivation may lead us to seek certain kinds of movement in a painting); genuine art will not cater to emotions that take the concrete

conditions or goals of the individual's own specific life situation as their object, such as whether he will eat dinner tonight. This does not imply that the artist may not _refer_ to the objects of concrete emotions in order to produce his abstraction, but rather that the concrete emotion is not the thing of ultimate value in a work. Such a notion is implied by Suzanne Langer's theory of feeling in art,[4] and by Eugene Gendlin's further development of Langer's theory,[5] in which Gendlin suggests that the object of feelings are not physical objects, but rather are 'felt meanings'.

We shall explore this possibility along three lines: (I) the ontological problems raised by the decoration-illustration distinction in relation to the expression of feeling; (II) the implications of the dynamic nature of art; (III) the relation of form and content in the referentialism problem.

I

Two interrelated issues are at stake in the referentialism controversy. (1) Does the work of art refer beyond itself for its full range of meanings and values, or are its meanings and values to be found entirely within its perceptual structure? (2) Is the meaning of a work of art to be assessed in terms of its _emotional_ effect on the spectator, or is this effect irrelevant to its value as art? Bell's view would imply that, if we define 'emotional' in terms of the moods and passions we experience in everyday or 'real' life, then emotion is irrelevant to art. But if we allow that there may be one special class of emotions (the 'pure, aesthetic' emotions) that pertain only to the appreciation of art and not to everyday life, then the thesis that art has a kind of 'emotional' impact would not be entirely inconsistent with Bell's view (as William Bywater suggests[6]), although such 'emotions' would have nothing to do with the ordinary emotions of life. In this respect, Bell's thinking is not altogether dissimilar to Schopenhauer's. Schopenhauer distinguishes between 'beauty' and 'interest' in art: Art's true value, its beauty, he says, lies in its expression of a clear _idea_, whereas human emotions are merely brought in as a necessary evil in order to sustain the human spectator's interest.[7] For Bell, as for Schopenhauer, the role of emotion in literature is misleading because of the extent to which literature must utilize extra-aesthetic factors in order to sustain the reader's interest. That literature must use human emotions to produce its spectacle does not imply that the emotions are essential or even relevant to the _artistic_ merit of the work.

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In attempting to clarify whether, to what extent, and in what respects art has as its purpose the 'expression of emotion', it will be best not to ignore certain ontological problems. Many of these have been well summarized by John Hospers in the following questions |8|: (1) Whose emotions are to be conveyed? The artist's? If so, how should he select which emotions of his to convey? Does he convey the emotions he feels while actually writing the work? Of what interest are these to the spectator? (2) What does the word 'express' means in the thesis that art expresses emotion. If it means, as Tolstoy suggests, that the spectator must feel what the artist himself has felt, the problem arises as to how a listener can appreciate the melancholy of a Mozart Andante without himself feeling melancholy while he appreciates it. But if 'expression' means that the spectator simply understands and acknowledges what the artist has felt, then a mere expository account of the artist's feelings would suffice--in which case the grumbling of a chronic grouch should also qualify as art.

If Tolstoy's view is that art expresses the artist's own emotions, we might also ask him a third question. There appears to be such a phenomenon as 'self-indulgence' in writers and actors who use such techniques as 'cheap pathos' to sweep the audience along in a torrent of irrelevant or unmotivated emotion. Critics generally frown upon such 'self-indulgence', apparently with some justification. Though a technically-gifted artist may succeed in inducing his audience to 'appreciate' a self-indulgent display, may it not be that the appreciation they experience is more an appreciation of the artist's gift of illustration (i.e., the illustration of emotions) than of his accomplishment as an artist in the more restricted sense of 'true art' as opposed to mere illustration or mere decoration? The thesis that art must be distinguished from mere decoration and from mere illustration, of course, does not imply that illustration and decoration cannot attain to the status of art; it implies rather that the features that determine whether they qualify as art are different from the features that qualify them as good decoration or good illustration.

Referentialism sometimes seems to impy that the ultimate purpose of art is to communicate emotions which take the phenomena of the individual's own life as their object. This would entail the absurd conclusion that the speeches of a psychotherapy patient or the temper tantrums of a child should be classified as art. But the non-referentialists ignore the equally important point that the emotions of art are based upon the

emotions of life and that art must therefore refer to
these for its ultimate value and meaning. The truth
must certainly lie somewhere between these extremes.
Good art, whatever else it may accomplish, must at
least engender feelings which abstract from the par-
ticular emotions of life. These feelings are no diff-
erent from the emotions of life except that they take
the abstract rather than the concrete as their
object.[9] To move an audience into this realm of
emotional abstractions, however, art must refer beyond
itself to the full range of subjective moods and value-
feelings. The danger that referentialists like Tolstoy
flirt with--the danger of failing to distinguish con-
crete from abstract emotional content--can therefore be
avoided within a modified referentialism, attenuated
with the qualification that only abstract or univer-
salizable emotions are to be 'expressed'. (This, of
course, does not imply that the <u>characters</u> in a novel
cannot feel concrete emotions, or even that the reader
cannot empathize with them, but only that this empathy
is not the ultimate goal of a good novel.)
    This thesis would provide a convenient answer to
Hospers' question about <u>whose</u> emotions are supposed to
be expressed. Neither the specific emotions that the
artist feels in his everyday life nor those the spec-
tator ordinarily feels should be elicited. (What point
would there be in eliciting emotions someone ordinarily
feels anyway?) Rather, emotions whose objects exist no
more for one person than for another--emotions with
abstract, universalizable objects--are the central
point of interest. Of course, particular, tangible ob-
jects must be used as part of the artist's tool kit in
eliciting these emotions, but the particular object by
itself is not the thing toward which the emotions are
directed; it merely represents a more general constel-
lation of values and truths which is the true excitor
of the aesthetic emotions.
    Art is more concerned with evoking emotion than
with depicting it. To depict in a painting the emotion
of a mother crying because her son has died is
illustration; to paint the same subject in such a way
as to evoke in the viewer a mood of much broader scope,
and of a different kind, is art. This different kind
of emotion, however, can by no means be independent of
the emotions we experience in relation to 'real life.'
Joshua Reynolds seems to have reasoned similarly when
he advised the aspiring artist to paint the general
idea rather than the particular thing, but to paint the
general as <u>found in</u> the particular.[10] At this point,
however, Reynolds tends to blur the distinction between
depicting and evoking. The depiction of emotion <u>can</u> be
one of the elements used in the evocation of emotion.

Whether an artist depicts the general or the particular
should be of little consequence, as long as the
response he evokes in the viewer is a general one.
Probably for this reason, Reynolds fails to see
Hogarth's greatness, criticizing him for choosing "low
subjects" for his paintings.[11] Certainly, no artist
can literally put abstractions on canvas, any more than
an algebraic equation can be played on the violin. But
if art is to evoke emotion, rather than merely depict-
ing or illustrating it, then the emotional response
which is art's aim must be in some sense a more ab-
stract kind of emotion than the ones we ordinarily ex-
perience in life; its object, at least, must be of a
more general nature than the simple tangibles relevant
to the usual, everyday emotional responses and desires.
The treatment of the subject is the object of the aes-
thetic emotion, not simply the subject in itself. This
treatment reveals the abstracting vision that makes its
emotional impact upon the viewer. However, these ab-
stract emotions are as dependent on concrete ones as
any abstraction is on its concrete exemplars. This
does not mean that abstract emotions do not themselves
exist concretely, but simply that they would not exist
were it not for the more concretely directed ones from
which they abstract.

At the same time that art must be distinguished
from mere illustration on the one hand, it must also be
distinguished from mere decoration or entertainment on
the other. Everything in the human environment evokes
some emotion or other--even if only confusion or
boredom--and therefore functions as good, bad or in-
different decoration. If we straighten a picture
frame, we contribute to the decorative quality of the
environment by changing the emotion it evokes from one
of chaos to one of harmony. The artist's aim in work-
ing his material is not so much to cause it to evoke
emotion (for this it already does), but to change the
emotion it evokes to one that he considers worthwhile.
If the finished product yields only a concrete feeling
of pleasure which like a shot of morphine refers to
nothing beyond the immediate enjoyment of the
sensation, it is decoration but not yet art. If on the
other hand it merely refers to something beyond itself
(for example, to some concrete emotion that the artist
has felt) but evokes no abstract emotion in the
spectator, then it in effect only calls attention to a
real-life phenomenon to which the spectator could have
reacted without the artist's help; it is therefore mere
illustration and not art. A work of art that is more
than decoration and also more than illustration must
both refer beyond itself to an abstraction not readily
apparent in real life and also evoke in the spectator

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an abstract emotional response which he would not or-
dinarily make to the concrete phenomena of life. The
essential distinguishing feature of art by contrast to
both illustration and decoration is that the object of
the emotion it evokes is an abstraction. Art is not
unique in addressing these abstractions (so does
philosophy, for example) but it is unique in aiming to
produce a directly emotional involvement with them.

The second part of this paper will explore the role
of both concrete and abstract emotional values in
motivating the aesthetic response to (a) dynamic art
forms and (b) static art forms. We shall find that,
the more genuinely artistic a work is by contrast to
mere decoration and mere illustration, the more dynamic
it is; therefore, no art form is truly 'static.' The
third part of the paper will show that form as well as
content can excite the emotions and that Bell's
distinction between form and content, upon which his
idea of 'significant form' depends, is artificial.
Almost any 'form' can become a content for some higher-
order form, and almost any content can be interpreted
as a form arising from the relationship of some more
basic contents. In order to have significant form, a
work of art must first have significant content in the
sense of an emotional meaning, though this meaning has
the abstract rather than the concrete as its object.

II

In assessing the role of human value and emotion in
art, it is helpful to clarify two distinctions, one of
which turns out to be a pseudo-distinction. First (and
this is the pseudo-distinction), dynamic art--art that
must structure itself in time--versus static art, which
need not be so concerned with time. Secondly, concrete
emotional values--those that appeal to issues that af-
fect the individual per se (such as whether he will eat
dinner tonight)--versus abstract emotions, which posit
more generalized or universalized values. All aes-
thetic value, however, whether abstract or concrete,
implies subjective value to a possible spectator.
(Whether it is limited to such subjective value alone
is another question, but it certainly implies at least
some subjective value to a possible spectator.)

Dynamic art--art that unfolds sequentially through
time--must have a plot structure or direction toward
which the spectator tends to want the story or com-
position to move. In the case of fiction, for example,
something must be at stake in the outcome of the plot.
In order to interest the spectator in the outcome, the
writer must motivate him to accept the issues at stake
as important, at least within the context of the story.

Action in drama, for example, must be 'motivated' in order to be effective. Unmotivated action makes no sense and only distracts from the meaning of the play as experienced. For an action to be 'motivated', the playwright must first have laid some groundwork for this motivation by setting the action in a context where the viewer has posited some values or ends toward which he wants the action to go, toward which he feels it 'should' go. If the viewer is not concerned with whether boy will get girl, then all actions in the play will appear unmotivated and therefore uninteresting. Dynamic arts must structure themselves in terms of ends experienced as emotionally charged with value. Drama, fiction, music and narrative poetry are obvious examples of such arts because they all elapse through time and therefore must set up a motivational structure to sustain the spectator's interest.

That dynamic arts involve human feeling does not imply that these feelings must be concrete. The viewer does not need to be in love with the heroine in order to want the hero to win her affection. He accepts this value by fiat in order to appreciate the story. He could as well decide that the heroine is a worthless creature undeserving of the role the playwright has given her. In this case, however, the playwright may be at fault for demanding that the viewer accept abstract emotional values which contradict real values he already possesses. Part of the writer's job is to avoid this problem by making the viewer feel that the heroine is a worthwhile character. To this extent, the play must appeal to concrete emotions the viewer has brought with him from real life. But beyond this point, the good writer can make us posit values which we do not uphold in real life: He can make us want to see Tristan and Isolde united although in real life their plight does not concern us in the least. A composer can make us want to hear tensions resolved to the tonic note although such resolution has no bearing on real life. He could not do so, however, without making use of what he knows about real human emotions in order to motivate us. No one would accuse Tchaikovsky of corrupting the purity of music by using the real emotions of sadness and anger in his Sixth Symphony. Quite the contrary, it would lose its entire meaning and interest if these emotions were deleted. Its structure would be unmotivated at every point. With some music, such as that of Bach, it is more difficult to apply such simple emotional labels as 'sadness' and 'anger'; we should remember, however, that even Bach spoke of a doctrine of 'affects' in music.

Clive Bell's argument against art as evoking mundane or everday human emotion clearly does not apply to

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those dynamic works of art which involve abstract
emotional direction. The paradox that we 'enjoy'
tragedy, though it also makes us suffer, is comprehen-
sible only because the emotion we experience is ab-
stract rather than concrete; we experience generalized
or hypothetical suffering.[12] But however abstract
this suffering may be, to remove the humanly-defined
values and goal orientations would be to render the
story uninteresting and to destroy the cathartic
effect. In the case of such dynamic art forms as drama
and fiction, the meaning of the aesthetic experience
obviously requires that the spectator have defined or
posited something beyond the mere form of the work of
art as valuable or valued.

How does Bell's argument apply to dynamic arts that
involve concrete values? Perhaps we should not regard
anything as art whose value would rest solely on con-
crete human values. Pornography, for example, trades
almost exclusively on the unsatisfied sexual desire of
the reader. Take away the sexual desire and he loses
interest as rapidly as the sated stomach loses interest
in food. In fact, a purely pornographic novel relates
to the reader's sexual desire in very much the same way
that an attractive restaurant menu relates to his
hunger. In either case, if the supposed aesthetic
value of the object depends solely on the corresponding
concrete desire, this 'value' disappears as soon as the
desire is sated. Bell's point remains valid in these
cases at least to this extent: ˙If we admire a menu
only before we have eaten, we admire only its content,
which appeals to specific concrete desires. The more
clearly the menu depicts this content, the better it
serves as illustration, as opposed to art. But if we
admire the menu after we have eaten, something other
than its concrete emotional content must appeal to us--
namely (Bell would say) its form. Illustration can be
art, but it need not be.

But at this point we must also bear in mind an im-
portant difference between the pornographic novel and
the menu. The novel is a dynamic art form which must
have a plot to sustain our interest and therefore must
have abstract human emotion to interest us once our
concrete emotion (such as sexual desire) has become
irrelevant. The plot of a novel can have no form
without a humanly-defined value structure. That is why
most novels have human beings in them. Thus, for a dy-
namic art to have what Bell calls 'significant form',
it must also have significant content in terms of
(abstract) emotional values to which human readers or
listeners can relate. Dynamic art defines its form in
terms of such emotional content and must do so in order
to sustain the spectator's interest. Even the most

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avid lover of art-for-art's-sake would lose interest in a novel with no characters or plot, just as the most avid sportsman would lose interest in a game with no goal-oriented structure. In fact, there can be no game without some such structure, any more than there can be a novel without humanly defined values.

But what about the seemingly 'static' art forms such as visual art and non-narrative poetry? Perhaps Bell's theory is true for them?

We must ask first of all whether visual art and non-narrative poetry really are static or whether the movement they imply presupposes or creates a motivational directedness in the viewer which is essential to his grasping the meaning of the work. For example, crossing lines in a picture frequently imply 'blocked' motion. The viewer in his kinaesthetic orientation takes up the direction of a line as soon as his eye is compelled to trace its path. He then perceives the interruption of his eye movement as thwarting this direction. A picture with many crossed lines usually creates tension for this reason. Straight lines enhance this effect still further because they impel the eye more definitely in one direction than does a curved line. The eye also tends to move from the thick end of a line to the thin end. Through such techniques the artist gives his pictures an 'impact', which we experience by contrast to dime store pictures which are almost completely static and therefore 'fall flat'. Even in the case of the 'static' art forms, the more artful examples tend to be dynamic. A vivid painting of a horse, says Maurice Merleau-Ponty, may show one foot in one instant of time and another foot in another instant, so that the effect of motion is achieved.[13] Even an effective still-life impels the eye to move this way or that, generating an emotionally-meaningful pattern. There is no truly static art. But, as in the case of the abstract 'suffering' in tragedy, the tension we feel in viewing a painting is abstract and does not need to appeal to the viewer's private emotions so much for its effect (which is not to say that the presence of such emotions, or some previous acquaintance with them, may not enhance the effect).

It is important to note that in the example of tension in a painting it is the 'form' of the painting rather than its content that evokes an emotional reaction. In the examples Bell uses, such as crucifixion scenes, we are led to believe that it is always the subject matter depicted in the scene that evokes emotions other than the 'purely aesthetic' emotion; Bell would have us assume that the form has nothing to do with ordinary or mundane emotions. By

failing to consider the relationship between abstract
and concrete emotional values, Bell ignores the diff-
erences between evoking emotion and merely depicting
it. He therefore concludes, erroneously, that to ap-
peal to the everyday emotions in any way whatever is
irrelevant to the purpose of art.

III

If a story or symphony is to be 'beautiful', it
must, as Bell suggests, have a good form or structure.
No novel could be beautiful if its tensions were
resolved ex nihilo or if the resolution of all
questions in the story were reached in the first chap-
ter and all the rest served as denouement. But since
at the root of the formal structure of a novel or a
symphony are human moods and passions, form and content
cannot so clearly be distinguished for a comparison.
The meanings of the words 'content' and 'form' have
been so hotly disputed in aesthetics for so long that
they might best be avoided, except that they are in-
dispensable in the context of the referentialism
dispute. Given this context, it is convenient to use
them in their essential referentialist sense,
especially since Bell himself, the arch-anti-
referentialist, seems to do so. By 'content' we mean,
roughly, the elements of a work of art which correspond
to the specific emotions evoked (for example,
revolutionary activity in a play corresponds to
revolutionary fervor in the audience); by 'form' we
mean the structural pattern into which the artist
weaves the content. It will soon become evident that
forms can also be used as contents for higher-order
forms, and contents are frequently built up from lower-
order forms. Also, form itself can be one of the con-
tents of a work of art if it evokes the 'purely aes-
thetic emotion' Bell talks about (although we shall
show that form can never be the only content in a work
of true art, as opposed to a work of decoration). In
spite of these limitations, the form/content distinc-
tion remains a usable one if we remember that the terms
have meaning only in relation to each other, and that
the form of a work may also function as part of its
content.
Non-referentialists like Bell seem to suppose that
a theory of art which casts human emotions in the cen-
tral role will mean that the painting of a crucifixion
scene, for example, will derive its artistic worth
solely or primarily from the emotions the viewer feels
regarding the subject matter depicted in the
representation. But this oversimplified view of the
emotions forgets that they can be aroused by the form

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of a painting--its texture, color composition, etc. It does not follow, however, that the mood associated with the form of a work can be independent of its content. Whether or not a realistic or semi-realistic scene is depicted will influence how the viewer sees the form: That a shadow should be almost as clear, as distinct, and as sharply outlined as the object of which it is the shadow means that the form of the painting contributes to the meaning of 'bright light', and the entire painting will then be viewed in terms of a light source; whereas if this same color composition could be viewed without being at all thought of as a realistic or semi-realistic representation of anything, but merely qua color composition (as Bell urges), the meaning of its form as perceived would be wholly different (it would lose the character of three-dimensionality; 'things' in the painting would no longer have the meaning of a sub-form as distinguishable from the sub-form constituted by the lighting-and-shading effects, etc.). Again, if the left sides of objects are bright, and the right sides are dull or dark, the contribution of this effect to the form of the painting will be greater than and different from the contribution that would be made if half the objects were brighter on the left, the other half brighter on the right. In these cases, the form of the painting as a phenomenon (i.e., as seen by a viewer) is different according to whether it is viewed as three-dimensional or two, whether light is viewed as directional or as a mere flood of light: in short, whether the painting is viewed as the representation of some possible scene or occurrence. To isolate that which is depicted from the way in which it is depicted, equating the former with content and the latter with form, is oversimplified and artificial. On the other hand, it is true to say that form determines mood, if we bear in mind that content (including the subject matter depicted, to the extent that it is relevant to eliciting the appropriate emotions) contributes to the overall meaning of a given form.

Not only whether realistic objects are depicted affects the form of a painting, but even what kinds of objects are depicted affects it. A rich carpet can be made to harmonize with washed out walls in a painting of a room; but if grass is painted with this same rich quality, while trees in the same scene are of a washed out color, something different happens to the form of the painting: A new relationship between and among elements of the composition arises--a relationship which has the meaning 'eeriness' or 'strangeness', resulting from the 'unnatural' color combination. Or again, if an object of special emotional importance,

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such as a person, is placed at the center of
attention--or perhaps an object which does not quite
'belong', such as a modern businessman in a Medieval
setting--, then the figure-ground effect will be shar-
per than usual. An object of special interest warps
the 'hodological space' of the entire picture. In
general, 'formal' relationships between and among
emotionally-charged 'content' elements can be used to
produce forms which are more emotionally significant
(on the abstract level) than the same painting might
have been if the artist had deprived himself of the use
of the emotionally-charged elements. To deprive him-
self of the potential for these kinds of relationships
would be like depriving his palette of the color red.
The meaning of El Greco's Toledo, for example, would be
quite different--and obviously less significant--if it
were done in gay yellows rather than in somber grays
and browns.

The relativity of form and content is still more
remarkable in the case of music. If we regard the form
of a musical composition as its overall structure, such
as sonata or fugue, then the individual parts--the
themes and the development sections--become the
contents. But if we regard a melody as a form, the
chord progressions and phrases that make up the melody
become contents. If we regard a chord progression as a
form, the chords become contents. But if we regard a
single note as a content, then a single chord can be a
form. Any element, it would appear, can be either a
content or a form.

Suppose we regard a single note as a content. The
simplest possible relationships among these content
elements would constitute what we might call 'lower-
order forms': harmonies, rhythms and melodic lines.
These simple, lower-order forms are already capable of
embodying rudimentary moods. The mood of a minor chord
can be distinguished according to emotional tone from
the mood of a major chord. A major chord with a major
seventh added produces still another mood. The wist-
fulness of Debussy's Reverie distinguishes itself from
the more heavy melancholy of a similar passage from
Tchaikovsky's Fifth Symphony partly because Debussy
frequently uses major-seventh chords, whereas
Tchaikovsky seldom does. A slightly higher-order form
emerges if we string together two or more chords, form-
ing a chord progression. The progression from a C
major-seventh chord to a B-flat major-seventh retains
the general emotional meaning of the major seventh but
deepens the meaning and adds new flavors or dimensions:
for example, among other qualities of this progression,
a notable one is the feeling of a deeper, more in-
trospective moodiness. A good example of its ap-

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plication occurs in the scene from Claude Lelouche's film Live for Life, in which the main character, an older man, goes to a party where young people are dancing and impressing each other with fashionable behaviors. As the protagonist sits and watches, thinking of the absurdity of such expressions of vanity, etc., his mood is at an opposite pole from theirs. To show how the scene appears from the perspective of the protagonist's mood, Lelouche deletes the sound from the scene, allowing only Francis Lai's wistful background music to be heard. The music captures the protagonist's mood primarily through the use of the chord progression from C major-seventh to B-flat major-seventh.

In this same music, however, Lai wishes not only to capture a mood, but to hint at an underlying passion--a value-laden and value-directed feeling--for the mood of the character stems from something he is learning about value in life during the course of the film. Lai embellishes his chord progression with two devices: A passing chord--B seventh--is inserted between the C major-seventh and the B-flat major-seventh to increase the feeling of directionality, of longing for resolution (a seventh chord so naturally functions as a 'leading-tone' chord that it is normally expected to do so); and this feeling is increased still further by suspending a fourth in the B-seventh chord (for a suspension does just what its name implies--creates a suspense or tension that longs for resolution).

This example illustrates several features of content-form relations in music: (1) The lowest-order forms embody moods. (2) Slightly higher-order forms, such as the one just discussed, intensify moods, further specify them, focus them more sharply, and begin to put them into context of a value-directed sequence of mood-shades, thus bringing the dimension of passion into play. (3) As each lower-order form becomes the content for a higher-order form, the dependence of the very content of the mood itself upon its context in the higher-order form becomes increasingly important. (4) In music, it would appear as a general rule of thumb that form is to content as passion is to mood, if by content we mean a lower order form which becomes a content for higher order forms.

But if we pursue this line a little further, we see that this apparent general rule is an oversimplification. Features (2) and (3) above already suggest that it is artificial to think of a mood outside of its context, or at least that the same musical denotation in one context will connote a different mood from the same denotation in a different context. Furthermore, every passion is also a mood, in the sense that it has a specifiable affective tone as well as a value-

direction. So when several elements (whether moods or passions) interrelate formally so as to produce a larger whole, this larger whole necessarily embodies a more comprehensive mood, though it does not necessarily embody any more comprehensive passion. This can be seen clearly at the end of a musical composition. Ordinarily, tensions are somewhat resolved at the end of a piece, in the sense of a Gestalt closure. In fact, in a composition experienced as 'good', the structure of the composition is usually such that the taste which lingers after the last note is sounded is not merely the taste of that last note, but rather a feeling for the meaning of the piece as a whole. To the extent that there are still outstanding passions that would tend to motivate some further direction beyond the end of the composition, we usually regard this as a fault in the structure of the composition. Beethoven, for example, often experimented with many different endings for the same symphony. Too long or too heavy a coda would cause the meaning of the coda, rather than that of the whole symphony, to linger; too short a coda would leave outstanding passions unresolved. From these observations, we can deduce that although passions are indispensable requisites for establishing the overall mood of a whole composition, it is this overall mood itself which is of principal importance in the composer's purpose. We would have been misled, then, had we assumed that every mood is as simple and as concrete as are those which usually occur to us as common or mundane examples of them. In fact, complex moods usually do not occur to us as examples precisely because only art can readily express them.

Not only does the formal structure of a composition depend on content, but to a great extent the content dictates and defines the form. The meaning of the form as experienced by the listener is built up step-by-step as each form becomes content for a higher-order form. Formal elegance can only result from an overall inter-relation of musical progressions that remains true to the emotional content of its elements. The supposed distinction between form and content is really only a distinction between lower- and higher-order forms.

Thus, to say with Bell that a work of art has 'signnificant form' is also to presuppose that it has significant content. What makes this content sig-nificant is that it has a specific kind of emotional meaning--a meaning which refers to emotionally meaning-ful abstractions rather than to emotionally-meaningful particulars. These abstractions are equally meaningful for everyone, since they transcend the particular desires, goals and conditions of the individual's own egocentric life situation. The values relevant to art,

therefore, are not the values which are derivative from specific drives, needs or wants, either of the artist or the spectator: They are not self-seeking values. They make their appeal on a higher emotional level, the level on which all individuals share in one universal condition and all perceptual objects share in universal significances as defined by the reactions of the human spectator. Only when art moves in this realm of abstractions does it simultaneously escape the everdayness of mere illustration of concrete emotions (as in the case of pornography and didacticism), and the scantly significant 'prettiness' of mere decoration, which is too static to motivate our sustained interest. (Again, however, decoration and illustration can attain to the status of true art, but only if they transcend their respective concrete functions.)

In conclusion, then, the role that human emotions play in the appreciation of art is indispensable, for without emotion art would lose its dramatic structure and its dynamic impact. Art, by contrast to mere illustration, is concerned with abstract rather than concrete emotions. However, the abstract emotions of art depend upon the concrete emotions of life for their comprehension. We must understand in a general way the range of emotions if art is to speak to us. Which emotions we have felt may be less important than that we have felt some emotions or other and are able to abstract from them. The essential point is that we be capable of feeling emotions that do not relate directly to the satisfaction of our own private needs and wants. Without such emotions, art would have neither meaning nor value. We must therefore grant to the referentialists that art must refer beyond itself--to the human emotional predicament in general--in order to have meaning. On the other hand, the form of art is such that the emotions involved become abstractions and universals. Art contributes by means of its form to a content that inherently points beyond itself, precisely by referring to emotions having the abstract as their object rather than concrete conditions of interest to the artist or' to the spectator as such.

NOTES

[1]Leo Tolstoy, _What is Art?_, trans. A. Maude (London: Oxford University Press, 1955), p. 125.

[2]Clive Bell, _Art_ (London: Chatto and Windus Ltd., 1928), p. 27.

[3]Bell, pp. 27ff.

[4]Suzanne Langer, _Problems of Art_ (New York: Scribners, 1957).

[5]Eugene Gendlin, _Experiencing and the Creation of Meaning_ (Toronto: Collier-Macmillan, 1962).

[6]William Bywater, _Clive Bell's Eye_ (Wayne State University Press, 1975).

[7]Arthur Shopenhauer, "On the Comparative Place of Beauty and Interest in Works of Art", _Complete Essays of Schopenhauer_, trans. T. B. Saunders (New York: Willey, 1942), pp. 43-53.

[8]John Hospers, "Art and Emotion", _Proceedings of the Fourth International Congress on Aesthetics_ (Athens, 1960), pp. 662-66.

[9]George Lipsitz, "Herbert Biberman and the Art of Subjectivity", _Telos_, Summer 1977, pp. 174-82, says that Biberman attempted to create an art which appealed to the emotions, but to 'universal' and 'transpersonal' emotional values. Our thesis is that all true art (i.e., art that is more than mere decoration or mere illustration) does this.

[10]Joshua Reynolds, _Discourses on Art_ (New York: Bobbs-Merrill, 1965), pp. 39-56.

[11]Reynolds, p. 36.

[12]B. J. Rosebury, "Fiction, Emotion and 'Belief': A reply to Eva Shaper", _British Journal of Aesthetics_, 19. 120-30, discusses this concept of an emotional response to a hypothetical stimulus as opposed to a response to a stimulus supposed to be real.

[13]Maurice Merleau-Ponty, "Eye and Mind", _The Primacy of Perception_, James Edie, ed. (Evanston: Northwestern University Press, 1964) p. 186.