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# *Plato on Hypothesis, Proportion, and the Education of Philosophers*

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## **Introduction**

In the *Republic*, Plato has Socrates call for an unusual curriculum for the future philosopher-kings of the *kallipolis*, including ten years of mathematical training as well as five of dialectic and fifteen of political training. It is puzzling that a philosopher should study the mathematical sciences with such intensity. After all, in our society a candidate for a doctorate in mathematics would hardly spend ten years in graduate school, and the students in the *Republic* are not even trying to be mathematicians. Myles Burnyeat points out the basic question: “How, we may ask, will knowing how to construct an icosahedron help them when it comes to fixing the price of fish in the ideal market or understanding the Platonic Theory of Forms?”<sup>1</sup> It would be misleading to say that a general study of mathematics is useful in such matters. Because it is challenging to explain why mathematics is so important to philosophy we may be inclined to wonder whether the mathematical curriculum does after all have a central place in Plato’s thought. I will contend here that in order to understand Platonic metaphysics properly we must take the proposed mathematical studies seriously.

Even if we come to see that knowledge of mathematics is necessary for the student of the Forms, ultimately that knowledge is insuf-

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<sup>1</sup> M. F. Burnyeat, “Plato on Why Mathematics Is Good for the Soul,” in *Mathematics and Necessity*, ed. Timothy Smiley (Oxford: Oxford University Press, 2000), 2.

ficient for learning the Forms and studying philosophy as we usually know it. But if the mathematical curriculum is necessary, then contemporary friends of the Forms will need to set off to spend ten years studying arithmetic, plane geometry, solid geometry, astronomy, and harmonics. Perhaps the great problem with our attempt to make sense of Plato's curricular suggestions is that, if taken seriously, they turn traditional philosophical training into a path with a grave omission. Nevertheless, Plato does take quite seriously the position that mathematics is integral to knowledge of the Forms (and thus to philosophy). It is best to acknowledge from the start that the suggestion that mathematics is essential to becoming a philosopher is paradoxical, and, as Burnyeat writes, "Platonism is a philosophy which is paradoxical by deliberate intent. It goes knowingly *para doxan*, against the common opinion of mankind."<sup>2</sup> When Plato introduces something wildly counterintuitive he attempts to draw certain ideas to our attention; here what is being flagged is that there is more to philosophical understanding than what one might expect.

Despite the appearance that the Platonic education is about mathematics during this decade and about the Forms during the half decade dedicated to dialectic, there is in actuality only one subject treated (though studied in different ways) by the entire curriculum.<sup>3</sup> What the Forms and the objects of mathematics have in common is that they are both about value. The future philosopher-king actually studies the object of mathematics (value) for more than just ten years; throughout the curriculum the process is concerned with turning the soul of the student toward value, toward the Forms. What is significant, then, about this decade is that when studying the mathematical sciences the student has the opportunity to make a twofold advance. First, the student will see sensibles in terms of mathematics rather than seeing mathematics as the sensibles. For example, after this decade of study one will see a pair of shoes as an instance of Twoness rather than seeing two shoes as itself what it means to be two. Furthermore, the student will begin to see the way in which proportion brings things on a continuum into a spe-

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<sup>2</sup>Ibid., 81.

<sup>3</sup>The objects both of *dianoia* and of *noesis* are the Forms. When the Forms are studied properly they are *noeta*; they are objects of *dianoia* when they are understood through images (*Republic* 510b).

cific harmony. The goodness of the pair of shoes in our example lies in the proportions of the other Forms (e.g., Usefulness, Comfort, Style, Durability, etc.) with which Twoness is blended. The comfort, style, and durability are factors of the ratio of length to width, length to height, dark coloring to light coloring, firmness of material to softness of material, and so on. Meanwhile, the second advance the student should make during this decade is to become motivated to inquire into the objects of true understanding, namely, the Forms.

What makes this notion slightly dubious is that not all mathematicians become students of the Forms. So, mathematical study is not a seamless protreptic to philosophy. Perhaps this is because mathematicians do not necessarily study the particular mathematical sciences in the order prescribed by the Platonic curriculum. Asserting a kinship among these studies, Plato demands that *particular* mathematical sciences be studied in a particular order. The series of mathematical studies that Socrates calls for begins with arithmetic, proceeds through geometry, solid geometry, and astronomy, and draws to a close with harmonics (*Rep.* 522c-531d, 537c-d). The task of the future philosopher-kings is to gather together these mathematical sciences into a unified idea of the association of all things. Plato writes:

That common thing that every craft, every type of thought, and every science uses and that is among the first compulsory subjects for everyone . . . [is] . . . that inconsequential matter of distinguishing the one, the two, and the three. In short, I mean number and calculation, *for isn't it true that every craft and science must have a share in that?* (*Rep.* 522b-c; emphasis added).<sup>4</sup>

The entities in our world are amassed as the objects of all the crafts and sciences. If, as Plato suggests, the crafts and sciences have a share in mathematics, then everything in our world “has a share in” mathematics. Therefore, mathematical training would certainly be useful for understanding our world. If we understand the fundamental relationship between mathematics and the world, then we will more easily understand how mathematical training draws a student towards being (*Rep.* 523a).

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<sup>4</sup>Trans. G.M.A. Grube, rev. C.D.C. Reeve in Plato, *Complete Works*, ed. John M. Cooper (Indianapolis: Hackett Publishing, 1997).

This paper will argue that mathematics is essential to philosophical understanding in three main ways: (1) knowledge of mathematics generally fosters a fuller understanding of the world because, according to the *Timaeus*, the world is literally constructed with mathematical entities; (2) studying mathematics gives one ample practice with the method of hypothesis, which is indispensable during the process of learning the Forms; and (3) knowledge of proportion is necessary for understanding the relationships that Forms have with other Forms. The first section will look to the *Timaeus* where Plato imagines a demiurge constructing the world and its soul out of mathematical entities. It is all too easy to fail to take these odd claims seriously, but I shall argue here that once interpreted the *Timaeus'* odd claims provide a framework for philosophical thinking about the universe. The second section will compare and contrast the mathematical and philosophical uses of the method of hypothesis in order to show the utility and limitations of that method for learning the Forms. In the final section I will argue that the knowledge pursued by the philosopher (i.e., the knowledge of first principles or Forms) can only be had in the guise of knowing their context (i.e., the unique place of a Form within the hierarchy of reality) and that understanding proportion facilitates such knowledge.

## Curriculum and Cosmogony

Plato has Timaeus tell a likely story of cosmogony. The theme of what is said is that the fundamental fabric of our world is mathematical. Timaeus describes the creation of the body of the world out of the natural elements: fire, air, water, and earth. Two features of this account stand out in particular. First, it is a demiurge that crafts the universe, and the world is a *kosmos*, a universe constructed of a variety of goodness realized by the imitation of the creator's model (*Tim.* 28a-29a). Second, using the realm of what always is for a model, the demiurge fashions the universe out of the most proportionate relation possible among fire, air, water, and earth (*Tim.* 31b-32b). In order to unify these elements into a three-dimensional universe the demiurge used the best possible bond, that is "one that really and truly makes a unity of itself together with the things bonded by it, and this in the nature of things is best accomplished by propor-

tion" (*Tim.* 31c).<sup>5</sup> The dialogue indicates a commitment to the notion that the demiurge modeled the sensible world upon what is always changeless and that using this model he "reproduces its form and character" (*Tim.* 28a-b). Given the account of the proportional bond of the sensible world, we can infer that the structure of the model is also one of proportion. So, the argument looks like this:

- (P1) The sensible world is bound together by the bond of proportion.  
 (P2) The sensible world is modeled upon the realm of the Forms.  
 (C) Therefore, the realm of the Forms is bound together by the bond of proportion.

Of course, this argument depends upon what it means for the realm of the Forms to be the model for the sensible world. It is safest to hypothesize that it would mean that the model (i.e., the realm of the Forms) is similar in some substantial way(s) to what is derived from it (i.e., the sensible realm). If this hypothesis is correct, then knowledge of proportion is indispensable for one with hopes of knowing anything about either this realm or the realm of the Forms. Not enough can be made of the importance of this for understanding Plato's theory of education. Understanding proportion is the goal of the mathematical curriculum. We shall return to the evidence for this in the final section.

*Timaeus* also reports that the demiurge constructed the world soul by mixing Sameness, Difference, and a concoction of changeless, indivisible Being, divisible corporeal Being, and what is intermediate between these two kinds of Being (*Tim.* 35a). The demiurge then made one mixture out of these three. Plato writes, "[The demiurge] redivided the whole mixture into as many parts as his task required, each part remaining a mixture of the Same, the Different, and of Being" (*Tim.* 35b). The mixture was eventually used up once the world soul was completed (*Tim.* 36b). The *Timaeus* provides a detailed account of the world including the soul in mathematical terms, but it is still unclear how knowing *about* the principles of value would actually make one a better person. Joan Kung claims that only the study of mathematics can teach us "the ratios and proportions among the

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<sup>5</sup>Trans. Donald J. Zeyl in *Complete Works*. Plato follows the Pythagorean notion that the most perfect kind of proportion is geometric proportion.

psychic parts that constitute virtue.”<sup>6</sup> This answer is similar to one proffered by Burnyeat. His main claim is that studying the mathematical sciences is good for the soul for two reasons: (1) because they provide the student with an understanding of objective value; and (2) because the actual content of mathematics fashions within the human soul the structures that exemplify Forms such as Justice and Moderation. However, in order to discern how knowing about a mathematical value, such as geometric proportion, will cause the instantiation of it in the soul of the knower it is especially important to consider the importance of mathematical proportion for Plato’s understanding of cognition rather than of virtue alone.

Since the soul is a mixture of Sameness, Difference and Being (*Tim.* 37a-b), and Plato adheres to the principle of like being known by like, cognition is, therefore, mostly a matter of cognizing how things are the same and different from one another. The ability to perceive analogical relationships is one type (among others) of this cognition of sameness and difference. Thus, mathematical cases of analogy/proportion are crucial because they are the purest case of analogy, and as such they are the paradigm for all analogies. Therefore, our apprehension of all analogies is based on our comprehension of mathematical proportion (cf. *Grg.* 465b6-7). Meanwhile, there are those who take the Forms to consist of nothing other than mathematics, i.e., sets of ratios.<sup>7</sup> According to the view that Forms are sets of ratios alone, apprehension of the Forms would be nothing other than apprehension of proportion (i.e., the sets of ratios). However, this view cannot account for how it is that the Form of the Good is the source of the being and essence of everything else, how goodness is the ultimate reason for the presence of certain kinds of things in the created world. Forms cannot be reduced to sets of ratios, since complete knowledge of a given Form (e.g., the Form of

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<sup>6</sup>Joan Kung, “Mathematics and Virtue in Plato’s *Timaeus*,” in *Essays in Ancient Greek Philosophy*, v. 3, edited by John Anton and George Kustas (Albany: SUNY Press, 1987), 332.

<sup>7</sup>See, for instance, Erik Ostfeld, “The Role and Status of the Forms in the *Timaeus*: Paradigmatism Revised?” in *Interpreting the Timaeus-Critias*, edited by Tomás Calvo Martínez and Luc Brisson, 167-177 (Sankt Augustin: Academia Verlag, 1997); and Lloyd P. Gerson, “Imagery and Demiurgic Activity in Plato’s *Timaeus*,” *Journal of Neoplatonic Studies* 4, no. 2 (1996): 1-32.

Ox) must include understanding of the place of such things (e.g., oxen) in the *kosmos* as a whole.

The position of those like Kung and Burnyeat appears to entail that knowledge of mathematics enables one to know the structure of a Form such as Justice and that in turn this causes one's soul to take on the proper structure such that it becomes just. This sort of view is so tantalizing because it attempts to offer the short route from knowledge of mathematics to virtue. Yet, even if virtue is the necessary result of knowledge of the Forms, complete knowledge of Forms requires more than quantitative understanding of a given Form. Something qualitative is also requisite for completely knowing a Form such that it will necessarily result in the instantiation of virtue in one's own soul. This qualitative aspect is the understanding of how the entity already quantitatively known is related to the Form of the Good. As a result, knowledge of mathematics is a necessary but insufficient condition for virtue. The *Timaeus* contains important clues about how mathematics leads one towards the realm of the Forms (and, therefore, to virtue), and it also helps us to begin to see that what the Forms and the objects of mathematics have in common is that they are all about value. But the *Timaeus* does not enable us to answer completely the "Why math?" question.

## **Dialectic and the Mathematical Method of Hypothesis**

The ten-year endeavor to see reality through the lenses of a mathematician prepares the future philosopher well because someone who understands the Forms must be capable of making hypotheses in addition to having an understanding of mathematical proportion. In this section I will consider the importance of the mathematical method of hypothesis, and in the next section the usefulness of understanding proportion will be addressed. During the decade of mathematical study, the future philosopher acquires, at the very least, extensive practice in the method of hypothesis. In the *Republic* it is alleged that the study of mathematics leads to the activity known as dialectic, which yields true knowledge as it furnishes knowledge of the Forms. Two paradoxes are at hand: (1) it is not yet obvious how studying the mathematical sciences leads one to dialectic; and (2) Plato criticizes how the students of mathematics use the method

of hypothesis (*Rep.* 510c-e). The implication of the latter seems to be that the five mathematical studies have one method while dialectic has its own. I contend instead that the “mathematical” method that Socrates criticizes is not particular to mathematics alone.<sup>8</sup> Surely, a non-mathematician could proceed using the “mathematical” method of hypothesis, which Socrates describes as beginning with some assumed starting point(s) and deriving certain things from there, trying to make what is derived consistent both with each other as well as with the initial assumption(s). In fact, there appear to be many such instances within the Platonic corpus. Let us take up the example found when Socrates begins his first speech in the *Phaedrus* with two assumptions.

First he assumes that each of us has two ruling principles: our inborn desire for (sexual) pleasure and our acquired judgment of what is right (*Phdr.* 237d). Second, he assumes a definition of *erôs*, namely, that it is “the unreasoning desire that overpowers a person’s considered impulse to do right and is driven to take pleasure in beauty, its force reinforced by its kindred desires for beauty in human bodies” (*Phdr.* 238b-c).<sup>9</sup> Socrates insists upon assuming a definition because in characteristic fashion he thinks that we must know what *erôs* is before we can know whether or not it is good for the *erômenos*. Of course, the position that is developed out of these starting points is doomed because it is derived to be consistent with the starting points, which are not established as error-free. In the *Protagoras* we see objections to the first assumption and in the *Symposium* to the second. Thus, we can see that not only the mathematician

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<sup>8</sup>In his review of Charles Mugler’s *Platon et la Recherche Mathématique de son Epoque* Cherniss criticized Mugler for making the so-called old mistake of asserting that what distinguishes the methods of the geometer and the dialectician at *Republic* 510-511 “is not so much their procedure as their sphere of application,” since at 511c-d Plato says exactly the opposite (“Plato as Mathematician,” *The Review of Metaphysics* 4, no. 3 [March 1951], 415-416). Plato insists that it is their methods that distinguish them because the synthesis (upward movement) of 511b belongs entirely to dialectic. It is on account of this that Plato declares the supremacy of dialectic over mathematics in *Republic* VII. I agree entirely that the upward movement is absent from mathematics, but it is misleading to declare as Cherniss does that the method of hypothesis used by the mathematician is grossly different from what the philosopher does.

<sup>9</sup>Trans. Nehamas and Woodruff in *Complete Works*.

can proceed in such a manner that leads to erroneous derivatives. Even though one can find abundant examples of this methodology in the discipline of mathematics, we can say that the method of beginning with some assumptions and deriving certain things from there, trying to make those derivatives consistent both with each other as well as with the initial assumption(s) is not specific to the discipline of mathematics. So, henceforth I will use the terms “mathematical” method of hypothesis and “scientific” method of hypothesis interchangeably. If this method is not particular to mathematics, then it remains to be asked: what is the “dialectical” method of hypothesis and how exactly does it differ from the method of hypothesis that is associated with mathematics? And finally what are the metaphysical consequences of Plato’s understanding of the philosophical method of hypothesis?

### *Comparing the Dialectical and Mathematical Methods of Hypothesis*

Whether one is searching for a theorem in mathematics or a first principle in dialectic, the actual method of forming some hypothetical claim is the same. Making a hypothesis is the first step in the process of proving something. However, while the method is the same for both mathematics and dialectic, the character of what is proved differs greatly. What results from the mathematical method is something qualified, while what the dialectical method yields is entirely unqualified. What follows necessarily from an unproved hypothesis (i.e., a mathematical working first principle, a definition) is a hypothetical result. Meanwhile, if one has traced a hypothesis back to an unhypothetical, true first principle through dialectical analysis, then it is a proven hypothesis (i.e., one which has been traced back to the true first principle of everything, the *archê*). Plato maintains that dialectic does not consider initial “hypotheses as first principles but as stepping stones to take off from, enabling it to reach the unhypothetical first principle of everything” (*Rep.* 511b).<sup>10</sup> The impor-

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<sup>10</sup>Of course, one can imagine, as Baltzly does, that Plato’s silence about what exactly the *archê* is makes it possible that “the movement of thought which is characteristic of dialectic can occur in relation to other unhypothetical principles as well” (Dirk C. Baltzly, “To an Unhypothetical First Principle’ in Plato’s *Republic*,” *History of Philosophy Quarterly* 13, no. 2 [April 1996], 157). Baltzly argues that we can find examples of two unhypothetical principles

tance of the ultimate first principle for Plato's metaphysics cannot be overestimated.

At *Republic* 510d Plato compares this method of hypothesis with that of the mathematicians. Of how the mathematicians work, he writes:

They make these ["the odd and the even, the various figures, the three kinds of angles, and other things akin to these"] their hypotheses and don't think it necessary to give any account of them, either to themselves or to others, as if they were clear to everyone. And *going from these first principles* through the remaining steps, they arrive validly (*homologoumenôs*) at a conclusion about what they set out to investigate (*Rep.* 510c-d; emphasis added).

Here we see clearly that Plato believes that in their deductions of theorems the mathematicians treat their hypotheses as if they were first principles. Surely if they were to compare their "first principles" to the dialectician's first principles they would agree that they are not the same, that is, that their hypotheses are not proven first principles. Euclid's definitions in *The Elements* are exemplary of the hypotheses that mathematicians employ. For instance, in Book I, Proposition I ("On a given finite straight line to construct an equilateral triangle") makes use of the "first principle" that a straight line is "a line which lies evenly with the points on itself" (Definition 4).<sup>11</sup> The word that Plato uses in the passage above, which Grube and Reeve translate "validly" (*homologoumenôs*), is an adverb from the present tense passive voice participle of the verb *homologeô*, the primary meaning of which is to agree. This is crucial because this is exactly what we see in our example from Euclid's first proposition. What Euclid demonstrates in proposition 1 is in agreement with the way he defines a straight line (definition 4), but that definition itself has not been proven or deduced from anything else. For a mathematician it is acceptable to make a hypothesis, such as what a straight line is, and

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within the corpus at *Parmenides* 141-143 and *Sophist* 251-253. The *Sophist* passage declares that some Forms blend and that some do not, while the *Parmenides* text pronounces that the One has a share of Being.

<sup>11</sup> Euclid, *The Elements*, vol. 1, 2<sup>nd</sup> ed., trans. Sir Thomas L Heath (New York: Dover Publications, Inc.), 241, 153.

then demonstrate things that agree with the core group of hypotheses that one has made. Mathematical hypotheses are like dialectic's first principles in the role they play within the discipline, but Plato's criticism of the mathematical method of hypothesis is separate from this function. Instead, it is a criticism of what mathematicians care about or, rather, do not care about. These students take up the very same objects of study as the philosophers, namely, things that are intelligible (here unqualified being), but the mathematicians do so without any *archê*. What Plato criticizes is the lack of unhypothetical support for what is derived.

Through the analysis of some concept or proposition one can work out what is presupposed by it, and a continuous analysis will lead one in theory to the *archê* or first principle of everything. However, the dialectician's analysis of concepts is a strange endeavor due to how difficult it is to provide an unhypothetical *archê*. Given this complexity, it is evident that to give an account of a principle is not as simple a business as defining a triangle, for instance. Let us take up an example of a partial analysis (obviously we cannot undertake a complete one, since the purpose here is simply to provide an example). If we take the number three as our *analysans*, then we need to break it up into its simple parts. Plato conceived of numbers as multiplicities of unit (*Rep.* 526a). So, multiplicity is part of the concept of the number three, and unit is part of the concept of multiplicity. In this case, three units are entailed (i.e., the concept of Threeness). This is an example of the partial analysis of a concept (in our case the number three) into its *analysanda* (constitutive concepts). This analysis of a concept into its constituent concepts immediately gives a deduction of a statement from other statements, namely, assertions of existence. Each statement is a premise, and based on what must follow from that premise a conclusion is derived. That conclusion is, in turn, a premise from which certain conclusions can be derived. In the case of our example:

(1) Numbers are pluralities of units.

From this the following conclusions can be drawn:

(2) The number three is a plurality of units.

From (1) and (2) we can conclude that:

(3) Where there is the number three there is plurality.

(4) Where there is the number three there are units, specifically three units.

These conclusions in turn presuppose certain things. If the implications in the Republic are true, then we should be able to continue our analysis until we arrive back at the first principle of everything.

So, we have seen that the method of hypothesis that is associated with mathematics differs from the dialectical method of hypothesis. This difference consists of the character of the results of the deduction that follows the initial hypothesis. What is deduced from the scientific method of hypothesis depends on a hypothetical first principle, while the dialectical method of hypothesis has as its *archê* an unhypothetical first principle. Consequently, the results of the scientific hypothesizing can only be utilized by one who accepts the axioms of the systems. Meanwhile, the product of the dialectical hypothesizing can be used without restriction because it rests upon a proven first principle, which as such must be universally accepted. Nevertheless, despite the qualified utilization of scientific method of hypothesis, it certainly trains the student for making deductions in dialectic. One could hypothesize that virtue is knowledge and then combine that hypothesis with the premise that all knowledge is teachable in order to claim that virtue is teachable. This kind of reasoning reflects the way that Plato claims that the mathematicians operate, namely, arriving “validly at a conclusion” without giving an account of the hypothesis from which that conclusion is drawn (*Rep.* 510c-d). Here the hypothesis is treated as an axiom, and resting here is the mark of the person designated in the analogy of the divided line as a thinker, that is, someone content with a life of *dianoia*. The dialectician, on the other hand, moves beyond the scientific method of hypothesis precisely because s/he is unwilling to proceed with the axiom that virtue is knowledge. Instead, the dialectician would insist upon investigating whether or not it is the case that virtue is knowledge. Let’s say for the sake of argument that the dialectician were to discover that virtue is knowledge, then s/he could proceed with a proven first principle (rather than an axiom) as the foundation for the argument. On account of this difference, dialectic accomplishes for mathematics what, as Burnyeat notes, mathematics cannot do for itself, namely, achieve the highest intelligibility.<sup>12</sup> This highest intelligibility is attained only through the attainment of the ability to give an account in light of the first principle. The dialectician,

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<sup>12</sup> Burnyeat, 41.

emphasized in the divided line analogy not just as a thinker but rather as one who understands, pursues *noêsis* instead of mere *dianoia*. Thus, our earlier question about how studying the mathematical sciences will lead one to dialectic is resolved now that we see that only one who lives the life dedicated to understanding (rather than just thought) will overcome the hypothetical and achieve Platonic knowledge.

What causes a great deal of confusion on this issue is that the term 'deduction' is used to refer both to what the mathematicians do with their hypothetical starting point (i.e., the downward motion)<sup>13</sup> as well as to what is done by the dialectician *after* the unhypothetical first principle of everything has been reached. When Plato uses the language of downward or reverse motion he is discussing the latter, i.e., what the dialectician does after the deduction. Some call this downward motion synthesis, but it is safer to find terms that Plato actually uses to describe such secondary movement. Having observed the confusion about how 'deduction' is used as a term, let us note what is being overlooked. The most pervasive aspect of the language in *Republic* 511 is direction: there is talk of stepping stones (*epibaseis*), moving in reverse (*hapsamenos*), proceeding downward (*katabainêi*), proceeding *up to* the first principle (*ep' archên iousan*),<sup>14</sup> and what is in the section *below* (*katô*). The bulk of the literature concentrates on upward and downward directional language, but the water can get murky once we try to make this language correspond to the directional language of deduction.

To sum up, what distinguishes the dialectician from the mathematician is the pursuit of unhypothetical first principles in the

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<sup>13</sup>See for instance, F.M. Cornford, "Mathematics and Dialectic in the *Republic* VI-VII," in *Studies in Plato's Metaphysics*, edited by R.E. Allen, 61-95 (London: Routledge and Kegan Paul, 1965). Ian Mueller, however, is more careful than Cornford with his language; the main problem I have with Mueller is not his understanding of deduction but his use of the language of synthesis and analysis. He describes deduction as synthesis, and it seems to me that this language misrepresents the salient difference between "the way up" to the *archê* and the reverse movement. See Mueller, "Mathematical Method, Philosophical Truth," in *The Cambridge Companion to Plato*, edited by Richard Kraut, 170-199 (Cambridge: Cambridge University Press, 1992).

<sup>14</sup>When *epi* is used with the accusative it usually indicates place (up to, as far as, upon, etc.)

place of merely “working” first principles. Plato has Socrates say that Glaucon does not think that mathematicians really understand the objects of their science “because they do not go back to a *genuine first principle*, but proceed *from* hypotheses” (*Rep.* 511d; emphasis added). The dialectician wants to step to, move on to, proceed *up to* the principle that rests on no hypothesis. The dialectician goes in that direction by deducing one statement from another using a definition. The relationships that exist among these statements are parallel to the connections that exist among the Forms. The dialectician traces something back to the *archê* precisely for the sake of envisaging the relation of the *archê* to everything else in the hierarchy of reality. Knowledge of these relationships is the dialectician’s ultimate goal.

### *Accounts, Definitions, and Context*

Plato’s dialogues consistently maintain that one with true knowledge of something will be able to give an account of it. In the early dialogues giving an account is repeatedly understood as giving a definition for something. But we must ask what it means to give an account of Forms, especially of the Forms that reign quite high in the hierarchy of Forms. Definitions surely cannot include what is to be defined in the definition; so too it is inappropriate to expect the account of the *archê* to be a definition. Since all things depend on the *archê* for their existence, nothing can exist that does not depend on it. Therefore, to define the *archê* would necessarily entail utilizing a concept that depends for its existence on the *archê*. So, trying to define the *archê* without utilizing either what is to be defined or what depends upon what is to be defined for its existence would be impossible.<sup>15</sup> As Aristotle writes, “For in regard to the first principles of science it is improper to ask any further for the why and wherefore of them; each of the first principles should command belief in and by itself (*Topics* 100a31-b20).<sup>16</sup>”

Instead it seems more reasonable that to give an account of any first principle will require not a definition but rather a context, that is, an account of the unique place something has within the hierarchy of Forms. One could compare this to knowing an address by

<sup>15</sup> Cf. *Meno* 79d-e.

<sup>16</sup> Trans. W.A. Pickard-Cambridge in *The Complete Works of Aristotle*.

knowing the relation one house has to a neighborhood. So, knowing a Form entails knowing its neighborhood. On this point two passages in the *Sophist* stand out in particular. First at 253b-c the Eleatic Stranger says that the most important kind of knowledge is required in order to show correctly “which kinds mix with which and which kinds exclude each other” and knowing “whether there are any kinds that run through all of them and link them together to make them capable of blending” is also requisite, and also one must know “when there are divisions, whether certain kinds running through the whole are always the cause of the division.”<sup>17</sup> Immediately following these remarks the Stranger says that “to divide things by kinds and not to think that the same form is a different one or that a different form is the same” takes expertise in dialectic (253d). The Stranger finally concludes with Theaetetus that “it’s inept to try to separate everything from everything else... The weaving together of Forms (*allêlôn tôn eidôn sumplokên*) is what makes speech possible for us (*Soph.* 259e). To dissociate each thing from everything else, that is, to break apart this weaving, is to destroy totally everything there is to say. To carry the neighborhood simile forward, trying to separate each thing is like trying to understand distinct neighborhoods without understanding that streets always connect one neighborhood to another. So much is at stake in these passages from the *Sophist*, and I will discuss their implications further in the next section. In the meantime, let us take up J.L. Ackrill’s remarks on the blending of the Forms that is mentioned here.

Ackrill is one of few commentators to give attention to this blending together of Forms.<sup>18</sup> He concludes that “to map out the interrelations of concepts (inclusion, incompatibility, and so on) is the task of dialectic.”<sup>19</sup> Plato’s notion that there are connections between some

<sup>17</sup>Trans. Nicholas P. White in *Complete Works*.

<sup>18</sup>See also Beryl Logan, “Philosophy and Sophistry: Plato’s *Sophist*,” *Eidos* 6 (1987): 7-19. Ackrill, however, emphasizes the importance of *Sophist* 259e for purposes that are distinct from my own. He argues that in Plato’s later work the theory of Forms is no longer a metaphysical theory but something like a linguistic one. He claims that Plato maintains that “there must be fixed things to guarantee the meaningfulness of talk” and that these fixed concepts “must stand in certain definite relations to one another” (J.L. Ackrill, “*Sumplokê Eidôn*,” in *Studies in Plato’s Metaphysics*, 206).

<sup>19</sup>*Ibid.*, 204.

Forms, but not between every pair of Forms, presupposes the existence of what Ackrill calls "concept-friendships or compatibilities" and "concept-enmities or incompatibilities."<sup>20</sup> Having suggested that to know a Form requires the ability to give an account of that Form's context, I want to utilize this notion of natural compatibilities and natural incompatibilities as one way to understand what it means for certain Forms to be blended together in such a way that each Form has its own unique context. We are still left wondering how the dialectician will accomplish the mapping out of all these interrelations. Certainly it shall require that any given Form be traced back to the *archê* using the dialectician's method of hypothesis. However, the use of the dialectical method of hypothesis is necessary but not sufficient for genuine understanding. Hypothesis, even as it is used by a dialectician, does not suffice to give the future philosopher the complete understanding of the interrelations of the Forms which is requisite for revealing the contextual account. It is the understanding of proportion that completes the dialectician's ability to grasp the blending of the Forms.

Having asserted that genuine knowledge comes from the use of hypothesis as well as an understanding of proportion, my interpretation flies in the face of Vlastos, as I have already noted, and to some extent also Cornford, who writes, "Plato realised that the mind must possess the power of taking a step or leap upwards from the conclusion to the premiss implied in it. The prior truth cannot, of course, be deduced or proved from the conclusion; it must be grasped (*hapsasthai*, 511b) by an act of analytical penetration."<sup>21</sup> By describing the acquisition of this truth as an act of analytical penetration Cornford distances himself from Vlastos' interpretation of dialectical hypothesis. Nevertheless, it seems to me that Cornford does not go far enough. He tantalizes us by saying that "*noesis* is an immediate act of vision; the ascent is made by one or more sudden leaps."<sup>22</sup> He points to the *Symposium* 210e as an instance of the apprehension of the first principle coming suddenly<sup>23</sup> and emphasizes the

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<sup>20</sup> *Ibid.*, 202.

<sup>21</sup> Cornford, 67.

<sup>22</sup> *Ibid.*, 73.

<sup>23</sup> *Ibid.*, 93.

mystical terms in which such apprehension is described,<sup>24</sup> But he leaves us wanting more. My suggestion counters Cornford's only to some extent because he suggests that this mental experience (*noesis*) might be knowledge of how to divide a given concept into its latently contained elements.<sup>25</sup> He emphasizes (and rightly so, as we have seen) that the most crucial ability involved in truly understanding the given concept is knowing how to divide it into these elements. However, I disagree with Cornford's version of how hypothesis yields new knowledge and suggest instead that the use of hypothesis must be supplemented. Something else that the future philosopher learns during the ten years of mathematical training augments the use of hypothesis, namely, understanding proportion.

### Proportion, the *Sumplokê Eidôn*, and the Hierarchy of Forms

Vlastos argues that Plato jettisons the method of *elenchus* in favor of the method of hypothesis because there are certain pieces of knowledge that "no elenctic badgering could have elicited."<sup>26</sup> The method of *elenchus* does *not* aim at bringing the student to the correct answer except insofar as it is a tool used to eliminate false beliefs and incorrect answers. But how do we get the right answers? Vlastos attempts to show that the method of hypothesis furnishes the dialectician with this new information. We have examined the method of deduction, and we have seen the way in which it reveals the context that a concept has. However, we should see by now that the dialectician cannot rely on the method of deduction alone to reveal any knowledge not already present in the dialectician. For instance, in our example of the concept of the number three we were able to deduce the concept's latent elements, but we were only able to isolate those elements through deduction because we already had knowledge of the Greek concept of number as multiplicity of unit. If one did not know what number signifies one would not be able to make much progress with the method of deduction. So, the method

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<sup>24</sup> *Ibid.*, 89.

<sup>25</sup> *Ibid.*, 69-72.

<sup>26</sup> Gregory Vlastos, "Elenchus and Mathematics: A Turning-Point in Plato's Philosophical Development," *American Journal of Philology* 109 (1988), 375.

of deduction can provide the dialectician with new knowledge but only in a limited sense. By this limited sense I mean that the method of deduction is not useful for teaching something with which one is completely inexperienced. It remains to be seen how exactly one discerns a Form's entire context and, therefore, understands that given Form.

Plato writes that those who have succeeded at both mathematical and practical matters "must be led up to the goal and compelled to lift up the radiant light of their souls to what itself provides light for everything" (*Rep.* 540a). Cornford reminds us that in this passage we see Plato embrace the language of the Eleusinian mysteries. He writes that this is appropriate because:

Initiation ended with the *epopteia*, the sight of certain sacred objects 'in a blaze of light', coming after a long process of purification and instruction (*legomena*) in the significance of the rites that had been witnessed (*drōmena*). So Plato's course of intellectual instruction by verbal discourse, in mathematics and dialectic, is a passage from darkness to light, and ends with an experience of a different order—a vision.<sup>27</sup>

It is quite plausible that Cornford is right about the adoption of mystical language, but the best he does to explain the path to this mystical experience of seeing the first principle of everything is to say that it is "an effort of intuition, for which *kathoran* is often used—an effort to see the unity in a number of things."<sup>28</sup> As tantalizing as mystical language can be in one way, in another it leaves us wanting more, and after championing the mystical guise of the *noetic hexis*, Cornford also leaves us wanting more. In an effort to sate this desire I suggest that mathematical training provides the dialectician with a reward that goes beyond simple mental exercise; this privilege is the opportunity to study proportion, the understanding of which is the second mathematical ability required for understanding the hierarchy of Forms. Proportion is the most important aspect of mathematics for the future dialectician. This is certainly reflected in the order of the five mathematical studies that Plato prescribes in his curriculum, which culminates in the study of harmonics.

<sup>27</sup> Cornford, 93.

<sup>28</sup> *Ibid.*, 87.

While section one established the importance of understanding proportion for knowing our physical world and section two showed how hypothesis helps to furnish knowledge of Forms, this section will explain how proportion supplements the method of hypothesis in yielding knowledge of a given Form. To put the point of this section directly, truly knowing a given Form requires one to know the way in which that Form blends (or does not blend) with all the other Forms, and these relationships are expressed by proportion. Surely this is an odd thing to say. It seems hopeless to say that Justice is three parts Goodness and two parts Beauty. No, this section cannot hope to be a cookbook of Forms. But proportion is important for understanding the blending of the Forms because of the hierarchical structure of the realm of Forms, meaning that the Forms closest to the top of the hierarchy (e.g., the Form of Justice, the Form of Beauty, the Form of Being) will be *more* blended with the Form of the Good than are the other Forms with which the Good is blended (e.g., the Form of a Dog and the Form of Bed). Let us examine this hierarchy in more detail.

### *The Hierarchical Structure of Reality*

After discussing the method of deduction as the upward and downward movements of the mind from thought to thought we should now have better sense of the centrality of knowing each Form's context, that is, how each Form is related to the others. Regardless of which Form is the *archê*, the first principle of everything, there will be certain Forms that are most related closely to it. The interrelation of the highest Forms is demonstrated by the fact that the Forms are said to owe their existence and being known to the Good (*Rep.* 509b). In the *Republic* Plato upholds the Form of the Good as the *archê*; this is clear from the statement that the Form of the Good is superior in rank and power to the Form of Being. The most telling thing he says about what this superiority entails is that even the Form of Being depends for its own being upon the Form of the Good. The product of what we have seen thus far is that the Forms stand in a definite relation to one another. This network of interrelations is the structure of reality; it is a hierarchy governed by an *archê*.<sup>29</sup> The

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<sup>29</sup> It would be the aim of a much larger project to address the issue of

*archê* is connected to all Forms as the governing factor of the entire hierarchy of reality. As such the first principle is what provides the structure for the hierarchy of Forms, the structure of reality itself. Given that the *archê* is the primary unhypothetical first principle, all of the Forms are related to it, and truths about them follow from a complete understanding of the *archê*. Plato describes the movements of thought that ensue from grasping the *archê* as follows:

Having grasped this principle, it reverses itself and, keeping hold of what follows from it (*palin au echomenos tôn ekeinês echomenôn*), comes down to a conclusion (*houtôs epi teleutên katabainêi*) without making use of anything visible at all, but only forms themselves, moving from forms to forms, and ending in forms (*Rep.* 511b-c).

The reverse motion described here synthesizes all of what is entailed by a given concept. The result of the collective movements of thought is, then, knowledge of the contextual web that exists among the Forms. This context is the unique place of a given Form within the hierarchical structure of reality. Thus, by using the dialectical method of hypothesis the dialectician has grasped the first principle, which promotes vision of the interrelation, the blending, of all Forms, which in turn will provides one with the ability to give an account of any given Form, that is, to name its unique context. Since such genuine knowledge is the goal of the philosopher, the means for achieving it, namely, hypothesis and the understanding of proportion, are of the utmost importance in the education of future philosophers.

### *The Importance of Understanding Proportion*

Plato has ordered the mathematical training called for in the *Republic* so as to reveal the psychological route to dialectic. The dialectician fortunately has extensive training in mathematics, and this, of course, includes study of proportion. We should keep in mind that the demiurge bound the elements of the *kosmos* together by using proportion and that the *kosmos* was created from a model. Plato writes,

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whether Plato ultimately thought that Unity stands atop the hierarchy or believed that it is the Good that reigns over the realm of the Forms, or whether Unity and the Good are one and the same.

"This, then, is how it has come to be: it is a work of craft, modeled after that which is changeless and is grasped by a rational account, that is, by wisdom" (*Tim.* 29a). Knowing that the world is modeled on the realm of the changeless, we can say with Plato that this universe reproduces the form and character of that realm. Of course, the world does not reproduce all the characteristics of the model. We have already established that the *kosmos* is unified by a proportional structure, and so it follows that the realm of the Forms must also have proportional character that unifies it.<sup>30</sup> As a result, when the dialectician pursues knowledge of the entities of that changeless realm, namely, the Forms, s/he will have to understand proportion. With that earlier experience of proportion in hand the dialectician will be in a position to utilize this prior training on the dialectical endeavors at hand.<sup>31</sup>

Cognitive scientists include the benefits of parallel experiences in their treatment of analogical reasoning. An analogy is the systematic relationship between two analogs, a "target" analog and a "source" analog. The target analog represents the new situation that one is trying to figure out, and the source analog represents the old experience that is being adapted and applied to the target analog.<sup>32</sup> For example, if I am in a relationship with a partner who became interested in me while in a committed relationship with someone else (source analog), then I can adapt and apply that experience to my new situation, namely, the one in which I am trying to figure out whether or not my partner will leave me for someone else despite our commitment to each other (target analog). However, analogi-

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<sup>30</sup> There are, at times, three proportions to bear in mind. Suppose Justice is a proportionate distribution of goods. This is one proportional aspect; another is that the Form of Justice itself has a proportional structure. Finally, and most importantly, Plato implies that the entire hierarchical realm of Forms itself has a proportional structure.

<sup>31</sup> There is a coincidence here that must be noted: the Greek word '*analogia*' has two basic translations. Primarily it means 'proportion,' but it can also indicate an analogy. The coincidence is this: we have been relying upon certain *analogous* features of mathematics and dialectic, but the content of mathematical science in question, which I believe crosses over to the dialectical science, is *proportion*.

<sup>32</sup> Paul Thagard, *Mind: Introduction to Cognitive Science* (Cambridge, MA: MIT Press, 1996), 78.

cal reasoning can be useless if one bases the analogy on superficial similarities. If, e.g., with our cheating lover example, I leave out the fact that my partner's former commitment was to someone who was cruel and abusive, then I may decide that the two situations are similar and that I should not trust my partner. However, if one takes note of relevant differences, then it is advantageous when one has "some previous experience with a domain but little general knowledge of it: Hence, analogies can be computationally powerful in situations when conceptual and rule-based knowledge is not available."<sup>33</sup> Again in our example I have some previous experience with my partner's ability to be faithful, but my experience is limited. So, I cannot claim to have general knowledge of it (if I did then I would not be in the position of trying to figure it out), and the analogy will be useful in estimating my partner's capacity for monogamy. In the case of the future philosopher-king, the harmonics studied during the decade of mathematical training is the source analog, and the target analog is the set of objects that express proportion, specifically both each individual Form as well as entire collection of the Forms.

Mitch Miller finds the sequence of mathematical subjects significant for an ascent to the study of primary objects that are no longer figures but rather expressions of ratio and proportion. According to Miller, the fifth study, that of harmonics, is concerned purely with ratio, and he takes the three main aspects of ratio revealed in *Republic VII* to be: first, that ratio is the inner structure of non-spatial as well as spatial entities (e.g., souls as well as soles); second, that there is a continuum of possibilities complicated with ratio; and finally, that the beautiful and the good are expressed by mathematically harmonious ratios.<sup>34</sup> We take from these three aspects of ratio that there is an intimate connection between Forms and numbers, specifically ratios of numbers.<sup>35</sup> Miller describes how ratio gives order to the Forms because wherever there is ratio a continuum between pairs of

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<sup>33</sup> *Ibid.*, 80.

<sup>34</sup> Miller, "Figure, Ratio, Form: Plato's Five Mathematical Studies," *Apeiron* 32 (1999), 84-5.

<sup>35</sup> Miller provides with this line of thought the most successful account of the sense in which the Forms are, according Aristotle's report of the unwritten doctrines, numbers.

relevant opposites is coimplicated.<sup>36</sup> Plato has Socrates point out that any opposites that are relative to one another (e.g., hot and cold) are on an unlimited continuum (*Phlb.* 24b). Thus, where there is ratio there is such a continuum because ratio picks out a point on the continuum, thereby imposing limit (*peiron*) on the unlimited (*apeiron*). One must envision the pair of relevant opposites underlying the continuum structure that is co-implicated in all ratios.<sup>37</sup>

Thus, the philosopher's target analog is, on one hand, an individual Form *qua* expression of a particular point on a continuum between a pair of opposites, and on the other hand, the philosopher's target analog is actually the entire collection of Forms insofar as each Form is an expression of one point along an intricate crossroads of various continua between pairs of opposites. Returning to our example of how a student will look at a shoe after ten years of mathematical study, we can say that the Form of Shoe is the spot within the hierarchy of reality where all of the following continua intersect: with respect to comfort will be the long/short continuum, the wide/narrow continuum, the tall/short continuum, and so on; with respect to style will be the dark/light continuum, rounded/angular continuum, and so on; with respect to durability will be the firm/soft continuum and so on. Within the network of all continua at any given intersection there is a particular Form. When assessed collectively these continua compose a vast network (or neighborhood, to use our previous language) in which each particular continuum literally is a street that intersects at various points with other streets, other continua. These blendings, the *sumplokê eidôn*, are the expressions of the continua between numerous pairs of opposites. As such each Form is an "intersection"; each Form is a unique amalgamation of the many other Forms with which it is blended.

Thus, genuine understanding of a Form is knowing the recipe for each of these blendings, that is, one must know the context of how this given Form fits into the proportional structure of the entire realm of Forms. So, proportion is the key to unlocking the mystery of the interrelations that exist among the highest Forms. For instance, the five great Forms of the *Sophist* are mixed in such a way that each is

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<sup>36</sup>Ibid., 85.

<sup>37</sup>Cf. Aristotle's doctrine of the mean.

known by knowing the proportions of each of the Forms with which it is blended. For example, the Eleatic stranger says that the Form of Being is different from all of the other Forms, and as a result, it has a share in the Form of the Different. So, understanding the Form of Being includes knowing that it is blended with the Form of the Different (*Soph.* 259b). That is, being different can be said of the Form of Being. However, the Form of Being does not only have a share of the Different. It is also blended with other Forms. For instance, we can imagine first of all that it has a share of the Form of the Same as well. Beyond that there shall certainly be other Forms with which it is also blended; the Form of Unity/Oneness comes to mind immediately, given that in order for the Form of Being to be one entity it must be blended with the Form of Oneness. Thus, understanding a given Form entails knowing each and every Form with which it is blended, and the dialectician's previous experience with proportion is useful for figuring out the constituents of each form, i.e., the continua between pairs of relevant opposites. So it is that the truest grasp of Plato's theory of education is the one that appreciates the centrality of proportion and hypothesis for understanding Plato's epistemology.

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