

FOCUS ON EXCEPTIONAL CHILDREN

ASSESSMENT AND PROGRAMMING IN MATHEMATICS FOR THE HANDICAPPED

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MATH ASSESSMENT AND PROGRAMMING FOR HANDICAPPED

As a first step in the exploration of the topic of assessment and programming in mathematics for the handicapped, a set of philosophic assumptions (or biases) of the author should be shared. These are assumptions regarding the nature of mathematics, the handicapped learner, and their interaction vis-a-vis instruction. Mathematics is a body of concepts that is organized in a logical, sequential, hierarchical system. Notwithstanding differences in the formal descriptions that currently define this system, its inherent order or structure remains invariant. At the same time, however, the manifestation of the system, as operationalized through human performance, will remain imperfect since it reflects the effect of the environmental experiences and systematic instruction.

This assumption regarding the universal nature of mathematics prevents one from considering such possibilities as "mathematics for the normal child" as opposed to "mathematics for the handicapped child." That is, the system of mathematics that each learner must master to some degree remains invariable. The logical structure of that system does not change for the gifted student or the student with developmental disabilities.

Handicapped children with learning difficulties in mathematics—whether described as mentally retarded, learning disabled, or emotionally disturbed—are characterized by a common problem. These children are not achieving the range of educational objectives in the manner or at an equivalent rate as the majority of their peers. Within all the psychomedical categories of handicap, substantial numbers of children present unique groupings of achievement and nonachievement. For these children, the most important educational decisions to be made are the determination of the range of educational objectives to be selected for instruction (strategy) and the specific instructional methods to be used to foster student achievement of those objectives (tactics).

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When the characteristics of the learner are considered in conjunction with the structure of the content, certain implications become clear. The logical order of instruction in mathematics must remain relatively constant for handicapped children regardless of the nature of their specific disability. Changes in this order would only result in magnifying the achievement disabilities of handicapped children. Within the constraint, the educational task remains one of determining the range of topics (objectives) to be considered for instruction and the instructional methods selected to facilitate achievement.

The above set of assumptions allows the author to view issues in the assessment and instruction of mathematics for the handicapped as an extension, refinement, and specification of the issues that have impact on the education of all children. In other words, these assumptions allow for the normalization of or for mainstreaming the problems of mathematics instruction for handicapped children.

PROMINENCE OF INSTRUCTIONAL OBJECTIVES

A generalized revolution in instructional planning has taken place in the last decade. This revolution is marked by the prominence given to behaviorally stated instructional

objectives in the process of educational planning. It is a generally accepted assumption that instructional objectives, in some form, should provide the basis for instruction and assessment. This assumption can be confirmed through examination of current educational literature, state educational plans, curriculum materials, and the syllabuses for teacher preparation programs. While its manifestation at the programming level of the classroom teacher is certainly incomplete, it is probably fair to state that the teacher who does not use instructional objectives in her classroom planning is considered outside the mainstream by the remainder of the educational establishment.

Some Problems

In view of the prominence given to instructional objectives in current approaches to instruction and assessment of all children, it is imperative that we examine some of the implications of their use for instructional planning with handicapped children. Goodstein (1974a) has proposed that conventionally formulated instructional objectives (e.g., Mager, 1962) have inherent limitations that can impede the teacher in the process of sequence or management of instruction. As instructional objectives become more precisely formulated, there exists a parallel need to formulate more alternative objectives. As listings of alternative objectives become more numerous, the ability of the teacher to use the listings of objectives to make sequencing decisions for curriculum planning becomes more limited.

Eisner (1967) has also pointed out that one of the biggest problem areas in the use of precisely formulated objectives is the sheer number of objectives that can be generated for any subject matter area. In special education, where the degree of specification sought may be well beyond that presumed necessary for the education of "average" children, the number of potential objectives generated can mushroom. This fact is attested to by the length of many current lists of instructional objectives in special education (e.g., Nofsinger, 1972).

Matrix Approach to the Specification of Instruction

As this author has previously proposed (Goodstein, 1974a), this problem, created by our need for specification in the process of generating instructional objectives, need not lead us to adopting the solution of reducing the level of specification. Rather, it should direct us toward

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construction of instructional systems that allow for the organization and management of all necessary specifications. This necessitates the creation of matrices for the display of those elements that impact the specificity level of instructional objectives (Goodstein, 1974a).

For example, in composing an instructional objective for mathematics instruction the following areas of specification might be considered: the stimulus situation created by the instructor (how the instructional task is presented), the manner in which the learner is required to respond, the learning requirement of the task (e.g., Gagne, 1965; Bruner, 1966), the mathematical topic (e.g., addition without renaming), and limits upon the range of examples to be used (e.g., three digit addends). While these areas of specification are not exhaustive of the existent possibilities, one could easily observe the multiplicative effect that differentiation within each of these areas could have upon the number of mathematics instructional objectives created.

The solution that this author and his colleagues at the University of Connecticut have adopted in the development of instructional objectives for mathematics instruction with handicapped children (Cawley, Goodstein, Fitzmaurice, Lepore, Sedlak, & Althaus, 1975) has been to sequence instructional objectives primarily by mathematical topic and to arrange all further specification within organizing matrices. This allows the instructor to retain the degree of specificity necessary in order to individualize instruction to accommodate the instructional needs of handicapped children. It also provides for the instructor a logical structure for the sequencing and management of instruction. In essence, it assists in preventing the trees from obscuring the view of the forest. Further elaboration of this system for specifying instructional objectives will be made later in this paper.

CRITERION-REFERENCED ASSESSMENT

Concurrent with the growth of the behaviorally stated instructional objective as an instructional planning tool was the development of a new measurement tactic for the assessment of achievement. This measurement tactic has become known as criterion-referenced assessment (Glaser, 1963). Criterion-referenced instruments are collections of items that have been selected to assess the instructional outcomes of specific instructional objectives. Thus, it may

be said that such items are referenced to particular instructional objectives. The use of the word criterion derives from the means of judging item performance. The use of absolute standards of performance (criteria) replaces standards derived from normative performance in the judgment of pupil (or item) adequacy. Readers who wish a more detailed discussion of the theoretical measurement framework underlying criterion-referenced instruments are referred to Popham and Husek (1969) and Glaser and Nitko (1971).

Briefly, criterion-referenced tests differ from norm-referenced tests, aside from their interpretation, in the manner by which items composing the tests are selected. Norm-referenced test items are ultimately selected on the basis of their statistical properties. Variance in performance among groups of learners is expected and sought. Additionally, items must adequately contribute to the discrimination of poor and able students. Criterion-referenced items are judged for adequacy only as to their content validity. A lack of variance in the performance of a group of learners can be expected when instruction is relatively effective or noneffective.

Problems with Norm-Referenced Instruments

As has been observed by Jones (1973), criterion-referenced instruments represent several improvements over norm-referenced instruments in the assessment of handicapped children. Exceptional children often perform at the lower extremes of the distribution of scores on norm-referenced achievement measures. Thus, the number of items that they can successfully master is limited. This can reduce the statistical reliability of scores of children scoring in this range. Also, the reduced number of accomplished items limits the diagnostic interpretation of the instrument for programming decisions that have to be made by the classroom teacher.

Since the selection of items for norm-referenced achievement measures is determined by their ability to distribute scores at any grade level over a wide distribution, exceptional children are expected not to be able to master a number of items. This negative selection factor mitigates against inclusion of a corresponding number of items that exceptional children could be expected to perform successfully. This is the case even when these items would be representative of the mathematics program for these children.

Perhaps the most important contribution of the development of criterion-referenced assessment strategies has been the intent to match assessment with instruction on a much closer basis than in the past. As has been alluded to earlier, handicapped children often require changes in strategy or tactics in mathematics instruction. This could include omission of certain nonessential topics or modifications of instructional procedures. Thus, norm-referenced instruments often may lack substantive content validity for handicapped populations. This should never be the case for true criterion-referenced instruments. The same objectives that are used for program planning and instruction should be used to generate assessment items.

The use of the word "true" in describing criterion-referenced tests suggests that the opposite condition "untrue" exists. Several instruments have recently appeared that purport to be criterion-referenced, since they do not contain normative data and have been developed from a set of instructional objectives. However, it is this writer's opinion that, unless these same instructional objectives are used by the teacher in planning and instruction, these instruments are not very useful. In fact, many of these instruments become the basis very quickly for locally imposed normative expectations for performance. Scores generated from such instruments become benchmarks for judging the relative achievement of children. Consequently, in too many instances the original intent of such instruments to assess only those objectives for which instruction was planned or initiated has become obscured.

CURRENT DIAGNOSTIC TESTS

Present assessment instruments available for assisting the teacher in mathematics programming can be imperfectly classed as either achievement tests or diagnostic tests. The presumed distinction between the two classes of instruments resides in the intent of the diagnostic test to determine causality in the determination of nonachievement. It should be pointed out that simply because a test author determines to call his instrument a diagnostic test, this does not necessarily endow it with diagnostic properties (Cronbach, 1970). Within either class of instruments, the tests may be norm-referenced or non-norm-referenced. (This writer considers a test to be criterion-referenced only when its underlying objectives match those used by the

classroom teacher for instruction.) Most current diagnostic tests in mathematics are non-norm-referenced. The one apparent exception to this trend is *Key Math* (Connolly, Nachtman & Pritchett, 1971). However, as will be pointed out later, the question as to whether *Key Math* functions as a diagnostic test is an open question.

Until recently the development of diagnostic instruments in determining the nature of arithmetical disabilities has been the province of mathematics educators. Mathematics educators have typically looked to the causality of disability as being determined to a large degree by the structure of the subject matter. This stands in marked contrast to the prevalent attitude of special education specialists to look toward the child as the first step in the determination of causality for disability (e.g., Johnson & Myklebust, 1967; Lerner, 1971; Frostig & Maslow, 1973).

The *Schonell Diagnostic Arithmetic Test* (Schonell et al., 1957) reflects the position of its authors that a diagnostic test provides an analysis of skills, not an assessment. It covers only whole number combinations. The test tends to be lengthy for both the learner and the teacher. No actual modules for remediation exist once the child has been "diagnosed." This absence of a linkage between assessment and instruction for most diagnostic arithmetic tests, while prevalent, is regrettable.

Reisman (1972) has also developed a mathematics inventory for diagnosis. Reisman's basis for both diagnosis and instruction is task analysis. However, the computational sections of the inventory do not lend themselves to diagnostic use because subskills are not carefully controlled within the problems.

The *Buswell-John Diagnostic Chart for Individual Differences: Fundamental Process in Arithmetic* (Buswell & John, 1925) provides for an individual analysis of a pupil's difficulty in the four basic operations with whole numbers. The problems are arranged according to difficulty within each operation. The teacher has a checklist of errors which is used as the child orally explains his method of solving the problems. The test is not excessively long and is relatively easy to analyze for sequence of processes. This analysis, however, could be refined to help the teacher more accurately pinpoint where remediation is necessary. No systematic approach to the remediation process is given.

Key Math (Connolly et al., 1971) lends itself more to grade placement from a set of norms developed with a population of average children, rather than diagnosis.

Failure on an item or set of items does not give the teacher a clear view of where diagnostic remediation should take place. Because of the normative nature of the test, substantial skill gaps exist between items arranged on any of its subtests. Additionally, failure to control the problems for the nature of the algorithm that the child might use to solve the problem creates difficulty in the interpretation of pupil performance for remediation. In fact, *Key Math* more closely resembles an individually administered achievement test than a diagnostic test.

Some additional problems regarding the use of *Key Math* with educable mentally retarded children were recently pointed out by this author (Goodstein, Kahn & Cawley, in press). The lack of items in the midrange of difficulty for the test causes many children to reach ceiling quite rapidly. This causes the test to lose its power to discriminate performance changes for many handicapped learners who make slow progress through those topics covered by midrange items. In fairness, this tends to be a characteristic of many standardized instruments that were norm-referenced by the performance of average children and subsequently used with handicapped achievers.

Of course, there also exist numerous standardized mathematics achievement tests or subtests of achievement batteries. These instruments tend to be less than useful in the assessment of handicapped children for instructional decision-making. As was pointed out during our discussion of criterion-referenced assessment, norm-referenced instruments largely lack content validity for the mathematics instruction for handicapped children and, perhaps more seriously, have structural inadequacies (e.g., limited range of appropriate items) that seriously limit their usefulness.

At this juncture, the author wishes to alert the reader that what may be perceived as an extremely critical comment regarding currently available assessment instruments in mathematics does not imply that their use is totally without merit. To the contrary, any information regarding the achievement of handicapped children in mathematics is desirable in contrast to absolute lack of such information which exists in many educational programs for handicapped children. What it does suggest, however, is that substantial room for improvement does exist, especially in the area of the development of diagnostic tests and inventories. Some suggestions for direction that such improvements could take will be offered later in this article.

PROJECT MATH

Little systematic attention to the improvement of mathematics instruction for handicapped children was given prior to this current decade. Recognizing this lack of attention, the Bureau for the Education of the Handicapped funded Dr. John Cawley and his associates at the University of Connecticut to begin a systematic inquiry into the nature of mathematics achievement among handicapped children and, subsequently, to develop a mathematics curriculum known as Project MATH (Cawley et al., 1975).

An understanding of the instructional system for Project MATH is necessary in order to clarify the assessment tactics that were adopted. It is hoped that these assessment tactics will provide a basis for the remainder of the discussion.

Project MATH has been described as a *multiple option curriculum* (Cawley, 1972). This label is derived from the multiple components that comprise the curriculum that are presented as optional instructional tactics for the teacher as well as the multiple approaches to instruction contained within each component. The major components of the curriculum include instructional guides and correlated activity books for the directed instruction of mathematics topics, a verbal problem solving component that provides instruction and practice in mathematics through practice in information processing, and a laboratory component that extends mathematics learning into the area of application in social problem solving contexts.

The instructional guide component includes directed activities in six major areas (strands) of mathematics learning: Sets, Patterns, Geometry, Numbers, Fractions, and Measurement. Each strand contains a careful developmental sequence of mathematics concepts carefully selected to balance the integrity of mathematics content system with the mathematics needs of handicapped children. Thus, topics which are unessential to the logical development of mathematics understanding and socially irrelevant for handicapped children were omitted. Topics deemed essential but difficult because of present usage of terminology or symbols were carefully revised in order to reduce unnecessary complexity, but the essential mathematics concepts were retained.

The Interactive Unit

For each concept within a strand, differentiated instructional tactics are articulated through a system for speci-

fyng instructional objectives. This system has been labeled the Interactive Unit. An early version of the Interactive Unit has been presented by Cawley and Vitello (1972) and was subsequently revised based upon the field test of Project MATH materials. The Interactive Unit focuses independently upon instructor and learner behaviors. All instruction is classified according to one of four instructor behaviors and one of four learner behaviors. The four instructor behaviors are construct, present, state, and graphically symbolize. The four learner behaviors are construct, identify, state, and graphically symbolize.

Construct behaviors imply the active manipulation of pictures or objects (or personal movement) to create the primary instructional stimulus for the child or to define the primary response mode of the child. Present behaviors imply the presentation of a fixed visual display of pictures or objects, where the manipulation of that display is not crucial to defining the stimulus for the task. Present behaviors essentially deal with nonverbal stimulus materials. The teacher creating an instructional situation using a set of pictures in a book, a fixed display of objects on a table, or the presentation of picture cards would be in all cases engaging in present behavior.

Identify behaviors imply making an instructional response by choosing the correct answer from a range of possible choices. Identify responses can be made by marking, pointing, or vocalizing (e.g., by reference to position) the correct response. Identify behaviors essentially are made in response to nonverbal stimulus materials. State behaviors imply the creation of primarily an oral stimulus to the instructional task or the requirement for an oral response to a problem. Graphically symbolize behaviors are inclusive of most written or drawn instructor and learner actions. They are also extended to include (on the instructor side) written or symbolically drawn text material or such work as might have been previously prepared on a blackboard. Graphic symbolic responses would also include multiple-choice responding where the choices were primarily verbal or graphic in nature.

Space does not permit elaboration of the implicit set of rules or procedures developed by Project MATH developers to code various instructional tasks to the Interactive Unit. The operationalization of the Interactive Unit became a process of trading off elegance in the description of instructional interactions for simplicity and usability in curriculum development and instruction. This author fondly remembers the many hours of debate over interpre-

tations of the Interactive Unit. The point of these observations is that the Interactive Unit is a heuristic device of organizing instructional objectives. Its usefulness must be judged primarily by the effectiveness of its organizing structure for the differentiation of instructional tactics.

The four instructor behaviors and the four learner behaviors form a matrix that yield 16 unique combinations of instructional interactions. For example, if the mode of instructor behavior is construct, combining the four modes of learner response yields four unique patterns of instructional interaction: construct-construct, construct-identify, construct-state, and construct-graphically symbolize. Each pattern of instructional interaction provides the basis for development of the mathematical concept using a different instructional tactic. In other words, for each concept the Interactive Unit facilitates the potential development of 16 alternative objectives (tasks) for instruction.

Project MATH Assessment Needs

Project MATH instructional guides, being developed off the Interactive Unit, encourage the teacher to conceptualize her programming at two distinct levels of analysis. At the strategic level, the teacher must determine the concepts within the strands that she should select for instruction for any particular child. Children who have educational handicaps differ widely in their specific instructional needs. Many of these children receive special education assistance quite late in their educational experience. For many children there simply may not be enough time to develop the full range of mathematics concepts that the child might have been initially capable of mastering. For other children, specific areas for instruction (e.g., computational skills) may have been overlearned, but attention must now be directed toward the development of supporting concepts.

Superimposed over such strategic considerations are the tactical questions as to which instructional strategies should be adopted for a particular child. Proponents can be arrayed on either side of the argument as to whether a child should be instructed to his weaknesses or from his strengths. Perhaps, certain patterns of interaction should be selected because of their effect upon the affective development of the child in addition to the usual cognitive determinations. The Interactive Unit does not prescribe tactics; it merely exposes the options systematically to facilitate teacher decision-making.

Project MATH Concept Inventory

To assist the teacher in the determination of strategic questions regarding the selection of concepts for instruction, Project MATH has included a criterion-referenced instrument called the MATH Concept Inventory (MCI). The concept inventory includes one item to assess the major concept outcome for the series of instructional guides that have been developed for each mathematics topic in the curriculum.

Specifically, the inventory is intended to serve two major purposes:

1. It may be used as a *screening device* to assist with the placement of children in the curriculum.
2. It may also be used as a *mastery test* to evaluate student progress *after* a sequence of instruction has taken place.

Since the instrument is both criterion-referenced and an inventory, a significant amount of teacher flexibility is incorporated in its administration. The teacher may elect to begin assessment with any item. Additionally, the teacher might elect to give some or all of the items, in one sitting or over several sittings. The constraints of formal test administration, necessary to ensure the validity of norm-referenced instruments, are unessential when individual child mastery information for specific concepts is being sought.

In terms of its use in the strategic decision-making process, the inventory is but one tool for the teacher. It is not designed to be a replacement for teacher judgment. Unfortunately, many teachers look to assessment instruments for rigid prescriptive rules in an effort to replace the need for effective decision-making. This is especially dangerous in regard to instructional planning for handicapped children. Handicapped children have such diverse patterns of cognitive and affective needs that linear systems for making programming decisions are not sufficiently comprehensive for effective and efficient instructional planning.

In respect to the MCI, when a child "fails" an item, this information is prescriptive for placing the child at some point in the corresponding sequence of instructional guides. However, when a child "passes" an item, several reasons might still compel the teacher to place the child in that sequence of instructional guides. Social and language

outcomes are also important goals of instruction with Project MATH. Often, instructing a child who has little difficulty with the mathematics concept being developed in the guides allows the teacher to emphasize equally important nonquantitative cognitive and affective outcomes.

Additionally, because the items for the concept inventory were designed as multiple-choice items, the items reflect only the use of the six cells of the Interactive Unit that provide for that manner of responding. Specifically omitted were any items that would require a construct behavior on the part of the teacher or construct or state behaviors on the part of the child. Thus, although great care was taken in the selection of an item that was representative of the concept, the child might yet encounter some difficulty* when performance would be required in alternate cells of the Interactive Unit.

Lastly, teachers operate in many instructional environments. Even if the teacher had an instrument with ultimate levels of precision in directing instruction, the teacher may determine the necessity for instruction within groups. This determination could be a result of teacher choice in regard to overall strategy, or it might be forced upon her by the environment that creates logistical problems to totally individual programming. For such teachers, grouping decisions become trade-offs between diversity in performance among individual children and a general level of achievement among a group of children. Thus, certain children might have to participate in instruction over concepts that they have demonstrated mastery in order to maintain instructional groups. This author does not view this tactic as inherently "bad," provided the teacher uses this opportunity for enhancement of other learning outcomes with these children.

One might raise the question as to the comprehensiveness of the strategy of designing only one inventory item for each concept. Certainly, reliability of performance on any single concept is severely impacted by this decision. However, in instrument design one is constantly faced with the choice between designing an instrument with enough items to ensure reliable performance and creating an instrument that is compact enough not to be a burden to either teacher or child. When designing the concept inventory, it was felt that unreliable performance would most probably impact our confidence in whether the child really has mastered the concept. Since nonmastery was the more prescriptive of the possible outcomes for any item

and the teacher was already cautioned regarding the prescriptive limitations of item mastery performance, the damaging impact of potential unreliability of item performance was minimized.

Project MATH Instructional Evaluation Items

While the concept inventory assists in decision-making at the level of strategy, an additional assessment tool is built into Project MATH for tactical decision-making. Each instructional guide, which has been developed to reflect a unique mode of teacher-pupil instructional interaction from the Interactive Unit, contains its own instructional evaluation item or items. These items assess the mastery of the instructional activities in the instructional guide within the same pattern or mode of teacher-pupil instructional interaction.

Figure 1 displays a facsimile of a Project MATH instructional guide. For this guide, the content strand is Numbers, the area is Cardinal Property of a Set, and the concept is Fewer Number/Greater Number. The input and output modes of the Interactive Unit are present and state. They are graphically depicted in the upper right hand corner of the guide. The behavioral objective is a simple description of the behavioral exchange summarizing information from both the Interactive Unit and the content descriptors.

The reader will note the consistency of the activities suggested on the guide in reference to the pattern of instructional interaction. Similarly, the evaluative items for this lesson guide reflect the same pattern of teacher-pupil interaction.

A record keeping system is provided for the teacher that allows her to record achievement for individual instructional guide evaluations as well as the concept mastery inventory performance for each sequence of instructional guides. Careful attention to such record keeping would allow the teacher to observe consistent trends in the effects of tactical changes in instruction. For example, a child may consistently demonstrate failure on all instructional guides that require a construct response. For such a child, the teacher *might* decide to delay instruction on all such instructional guides until some point later in the child's instructional program. Alternatively, the teacher might wish to plan more time for instruction for that child on such instructional guides. The diagnostic significance of the consistent use of such an assessment-record keeping system should be obvious.

Many writers have suggested elaborate strategies for the development of criterion referenced test items (e.g., Hively et al., 1973). Most suggestions focus upon the definition of domains from which items may be sampled. Evaluation items for Project MATH instructional guides follow a rather simple set of procedures. These will be shared in the view that they might be helpful to the teacher who might wish to develop her own assessment program.

First, the item should represent the terminal performance of any sequence of instructional activities on a particular guide. Second, the item should be consistent with the mode of teacher and learner instructional interaction reflected in the instructional guide. Additionally, items were viewed as reflective of two basic types. Type I items would sample the range of possible activities, where the set of those activities was finite. For example, if the activities were organized to practice addition of single digit numbers whose addends were less than ten, the items would be limited by the bounds of those activities. From that pool, a set of addition problems could be selected.

Type II items would assess those activities whose range of possible examples were infinite. For example, if the child (in fractions) were to be required to recognize a whole from the display of its parts, a variety of tasks or examples could be used. Evaluative items would use a different example from those examples in the activities, which shared all relevant and/or salient features. In fact, many items will share both Type I and Type II characteristics.


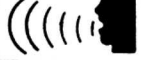
The validation of criterion-referenced assessment items requires (1) the careful selection of items related to any constraints imposed by the nature of the instructional objectives or the resultant instructional activities (similar to those imposed by the Project MATH Interactive Unit) and (2) the confirmation by the teacher that her impressions regarding student achievement during instruction are validated by item performance. This latter requirement, if not met, would require the teacher to review closely the item to determine whether unique features of the item are causing the discrepancy, resulting in a revision of the item.

It is recognized that the development of criterion-referenced assessment items and instruments may prove to be a difficult task for a great many teachers. This fact should only raise our commitment level toward the inclusion of such training in both preservice and inservice teacher education programs.

Figure 1

INSTRUCTIONAL GUIDE *

N 122

STRAND	Numbers and Operations	INPUT	OUTPUT
AREA	Cardinal Property of a Set		
CONCEPT	Fewer Number/Greater Number		

BEHAVIORAL OBJECTIVE	INSTRUCTOR	LEARNER
	Presents Pictures of Sets.	States whether a set has the same number, a greater number or a fewer number of items than a standard set.

ACTIVITIES

The instructor seats one or more learners on the floor or around the table. The instructor presents pairs of pictures of sets. The instructor asks the learner(s) to state if this set (holds one set higher) has the same number as this set (holds the second set up). If the response is no, the instructor asks, "Does it have a fewer number or a greater number?" The learner(s) responds in complete sentences, e.g., "That set has the same number". The position of the sets can then be changed and the questions repeated. Sets of different cardinal properties are used.

The instructor holds up pictures of sets of different objects. The instructor states that she can make up a story about the pictures (e.g., set of three boys, set of two baseball bats. There are a greater number of boys than baseball bats; there are a fewer number of bats than boys). The instructor then holds up two different pictures and asks the learner to tell a story. If the learner has difficulty doing this, additional examples should be given by the instructor.

The instructor includes instances when she holds up one picture of three balls and ask the learner if she has a greater number or a fewer number of balls. The learner must realize that greater number and fewer number are only meaningful in terms of two or more sets - they are comparison terms.

MATERIALS: Pictures of Sets

SUPPLEMENTAL ACTIVITIES: 122 a, b

EVALUATIVE CRITERIA

1. Use two pictures - one of five children; one of four ponies. Hold the picture of the ponies at a higher level than the picture of the children. Tell the learner to state which picture has the greater number. Switch levels of the pictures and tell the learner to state which picture has the fewer number.
2. Use the same pictures as above and tell the learner to make up a story about the two pictures. (Try to elicit a story that deals with the greater number of children in relation to the fewer number of ponies).

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Assessment Directly from Task Matrices

One other assessment approach manifested by the Verbal Problem Solving Component of Project MATH should be briefly discussed. Cawley (1970) and Goodstein (1974a, 1974b) have suggested that certain instructional tasks may be organized in such a manner that their organizational structure provides for both instruction and assessment to occur concurrently. For example, if verbal problems have been developed in such a manner that the various factors or parameters that combine to describe a problem have been controlled, organizational matrices can be used for retrieval of problems for both instruction and assessment.

For example, if a set of verbal problems has been developed from prescribed word lists at various levels, has defined information processing requirements, and has computational difficulties assigned to the problems in a systematic manner, any problem should be capable of description in relation to those three factors. An organizing three-dimensional matrix that arrayed reading difficulty levels, information processing requirements, and computational difficulty levels would provide a means of sampling problems for assessment as well as instruction. Unfortunately, at present there exist few subject matter domains where enough basic knowledge exists regarding the interaction of various subfactors upon performance to construct such management matrices.

THE NEED FOR A NEW DIAGNOSTIC MODEL

Earlier in the paper this writer alluded to the development needs in the area of diagnostic assessment in mathematics for the handicapped. Lepore (1974) reports a substantial percentage of bizarre or aberrant computational algorithms or approaches to the solution of computational problems among both mentally retarded and learning disability children. Many of these bizarre algorithms result in consistent failure on computational problems. This probably confirms the experience of countless special education teachers.

This has led to discussion among the Project MATH staff of the expansion of the current content X mode analysis model to include algorithm as a third parameter. Such a model would allow for more differentiated analysis

or diagnosis of mathematical disability among children with severe mathematics disabilities. To this author's knowledge no current diagnostic assessment instrument attains this level of diagnostic prescription.

Additionally, what is needed to complete the assignment is a set of remedial modules which would be referenced to particular cells of this enlarged content X mode X algorithm analysis model. Thus, once a child's idiosyncratic pattern of behavior has been identified, the teacher would be directed to use a particular module for remediation of the identified disability. The development of such a diagnostic instrument with its associated remedial modules remains one of the greatest current challenges to our development expertise.

SUMMARY

In review, this author has attempted to provide a philosophic base to the discussion of mathematics assessment and programming for the handicapped. Issues in the development of instructional objectives and criterion-referenced assessment were reviewed. A brief survey of current diagnostic tests and the norm-referenced approach to assessment was presented and critically reviewed. Examples of a system of criterion-referenced assessment for both strategic and tactical decision-making were provided drawing from the author's experience in the design of the Project MATH curriculum system. An alternate assessment approach for use with highly structured content domains, such as verbal problem solving, was briefly discussed. And, finally, suggestions for an area of great challenge and promise in the further development of assessment instrumentation were outlined.

In conclusion, this author wishes to reaffirm his belief that regardless of the precision obtained in the development of assessment instruments, it must be the classroom teacher who will remain the ultimate instructional decision-maker. Assessment merely organizes the data in a systematic manner. The weighing of the alternatives for instruction must remain with the classroom teacher who is ultimately responsible for the management of instruction. The effective training of teachers to become master decision-makers remains the great challenge of our teacher education systems.

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CLASSROOM FORUM

*Edited by Alwyn Holloway, Center Coordinator
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Several of my students are having much difficulty learning multiplication facts. I have used many of the usual methods of helping them learn the tables, but the students have not successfully learned them. Do you have any ideas that I might use in helping these students?

This problem is certainly not unique to you or to your students. Different children respond to different techniques—some to flash cards and drill, some to records. Others do not seem to benefit from any type of drill or repetition. I have a "system" that has been very effective in helping some children with this problem. Certainly, it can be modified in various ways. It just might be worth a try!

In order to ascertain which students need extra help, an initial survey should be given to all students in the class. On the test sheet, place facts from tables zero through nine. Repeat each fact several times so that there is a total of 100 problems. Explain to the students that they will have five minutes to work as many problems as they can. They should be encouraged to skip any problems they cannot work quickly and return to them if they have

time. Students who get a score of less than seventy or seventy-five correct need help.

Procedure

Make out a ditto sheet with 40 problems from tables zero through three for those students who scored very low on the initial survey test. Make another ditto sheet of 60 problems for students who scored around fifty, but who did not seem to know some of the facts from the tables zero through five. Use the original test sheet for the remainder of the students.

First Day

1. Return the test papers to the students. Show them how to make their own flash cards. Help them find four or five facts that they do not know and place each fact on a separate index card. After use each day, place the cards in a file box behind name divider cards.
2. Have a graph or graphs prepared with each student's name on it. Show each student how to mark his score on the graph paper. A different color pencil can be used for each student. A dot on the graph denotes the number correct out of one hundred.
3. Administer the test again. Give students who are working on tables zero through three and tables zero through five only three minutes to work on their sheets. Students working on the one hundred problem page should be allotted five minutes.

Each Day

1. Return the test papers taken the day before with the the number correct at the top.
2. Instruct students to mark their charts by drawing a line from the previous dot to the present dot each

day. For most students, progress (seeing the line on the graph go up) is a strong reinforcer.

3. Students study *their own* flash cards for 10 to 15 minutes silently. When a student knows his cards, he raises his hand, and the teacher listens to him recite. If he says them all correctly, he may read or draw quietly until the other students are ready, or until 15 study minutes have gone by.
4. Administer the test again. After about ten days, the problems on the page should be rearranged so that students do not begin to memorize the test.

Critical Areas

1. Each child must be on the appropriate test for him at the moment (zero through three tables, then zero through five tables, then zero through nine tables).
2. The teacher must check at least every other day to be certain each student is studying four or five facts that were incorrect on his last test.
3. The teacher should monitor students to be sure they are working as fast as they can on the test, skipping problems they have not yet learned.

Teacher Comment

After several days this procedure becomes routine and it is not difficult for a teacher to handle as many as 20 or more students at one time. Students really seem to enjoy this method of learning. Out of 148 students, I had only four who did not learn their tables in 10 weeks or less. All but one student really seemed to enjoy the process.

We wish to thank Ms. Lucy White, Resource Teacher, DeKalb County Schools, Georgia, for writing this column.