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# Longitudinal Comparison of Place-Value and Arithmetic Knowledge in Montessori and Non-Montessori Students

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**Abstract.** Understanding of base 10 and place value are important foundational math concepts that are associated with higher use of decomposition strategies and higher accuracy on addition problems (Laski, Ermakova, & Vasilyeva, 2014; Fuson, 1990; Fuson & Briars, 1990; National Research Council, 2001). The current study examined base-10 knowledge, place value, and arithmetic accuracy and strategy use among children in early elementary school from Montessori and non-Montessori schools. Children ($N = 150$) were initially tested in either kindergarten or first grade. We followed up with a subgroup of the sample ($n = 53$) two years later, when the children were in second and third grades. Although Montessori curriculum puts a large emphasis on the base-10 structure of number, we found that children from Montessori schools showed an advantage on correct use of base-10 canonical representation in kindergarten but not in first grade. Moreover, no program differences were seen in place-value understanding in second and third grades. Although Montessori children used different strategies to obtain answers to addition problems in second and third grades as compared with non-Montessori children, no program differences in addition accuracy were found at any grade level. Educational implications are discussed.

Mediocre mathematics achievement has been a persistent problem of the United States' educational system. On international comparisons of mathematical knowledge, the performance of U.S. students perennially lags behind that of same-aged peers in East Asia and much of Europe (Gonzales et al., 2009). Results of national assessments within the U.S. are no more encouraging; on the most recent National Assessment of Educational Progress, 26% of U.S. eighth graders performed at a level classified as *below basic* (National Center for Educational Statistics, 2013). These inadequate levels of mathematics achievement negatively affect both the national economy and individual college, career, and economic opportunities (National Mathematics Advisory Panel, 2008).

Ensuring that children acquire basic numerical understanding in early childhood is central to improving mathematics achievement in the United States. Early mathematical knowledge predicts rate of growth in mathematics (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004) as well as mathematics achievement test scores as late as high school (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Stevenson & Newman, 1986). Specifically, place-value and arithmetic knowledge are foundational for later mathematics learning (Kilpatrick, 2001; Mix, Prather, Smith, & Stockton, 2014). The present study examined whether the Montessori approach promotes a better understanding than other public-school approaches of three foundational aspects for later mathematics learning: (a) base-10 and place-value understanding, (b) ability to accurately solve arithmetic problems, and (c) use of base-10 decomposition, an efficient arithmetic strategy.

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## Literature Review

### Base-10 and Place-Value Understanding

A multitude of studies have found that place value is a difficult concept for young children (Carpenter, Franke, Jacobs, Fennema, & Empson, 1998; Cauley, 1988; Cobb & Wheatley, 1988; Fuson, 1986, 1988, 1992; Fuson & Briars, 1990; Ginsburg, 1989; Kamii, 1986; Miura & Okamoto, 1989; Resnick & Omanson, 1987; Ross, 1989, 1990; Varelas & Becker, 1997). It takes several years for children to develop an understanding of the base-10 system and place-value notation. Preschool-aged children are able to judge relative magnitude of multidigit numbers and map large number words onto written symbols. However, before formal schooling, most children think of numbers larger than ten as collections of units rather than as groups of tens and units (Mix et al., 2014). Children's understanding of the base-10 numeric structure is typically assessed with a block task (e.g., Miura, Okamoto, Kim, Steere, & Fayol, 1993), in which children are asked to represent two-digit numbers using a combination of unit cubes and ten-bars. Between kindergarten and second grade, children increasingly use both tens and units to represent two-digit numbers (Miura et al., 1993; Saxton & Towse, 1998). Thinking of multidigit numbers as groups of tens and units translates into later place-value knowledge, which is critical for more complex arithmetic operations (e.g., 27 + 14). Varelas and Becker (1997) found that the percentage of children who both traded correctly and correctly identified digits in the tens place on a written arithmetic task increased from 56% to 77% to 98% between second and fourth grades. Comprehension of base-10 structure suggests a deeper understanding of how numbers relate to each other and how numbers can be incremented by intervals greater than one, such as tens and hundreds. Both of these skills are useful when manipulating numbers. Kindergartners, for example, who represent double-digit numbers as a collection of tens and ones rather than as individual units, are more likely to use sophisticated addition strategies such as decomposition, which in turn is related to greater accuracy in arithmetic (Laski et al., 2014). In later elementary school, base-10 knowledge is related to accuracy on multidigit arithmetic problems (Fuson, 1990; Fuson & Briars, 1990; National Research Council, 2001).

The age at which children accurately use 10 blocks and unit blocks to represent two-digit numerals seems to depend, in part, on their instructional experiences (Fuson & Briars, 1990; Fuson, Smith, & Lo Cicero, 1997; Hiebert & Wearne, 1992; Miura et al., 1993; Varelas & Becker, 1997). For example, Saxton and Towse (1998) found that a practice trial in which the experimenter demonstrated how to use 10 blocks and unit blocks to represent double-digit numbers had a substantial positive effect on the extent to which young children used both 10 blocks and unit blocks to represent double-digit numbers. Further, a recent study found no differences between East Asian and American kindergartners' use of base-10 representations in children with less than one year of formal instruction (Vasilyeva et al., 2015), despite these differences being well documented at the end of first grade, after more than one year of formal instruction (e.g., Miura et al., 1993). The Montessori mathematics curriculum places great emphasis on base 10 and place value using a series of materials (e.g., golden beads, stamp game, bead frames) that highlight these concepts, even with children as young as 3 years (Laski, Jor'dan, Daoust, & Murray, 2015; Lillard, 2005; Montessori & Simmonds, 1917). Thus, it seemed plausible that differences in young children's understanding of base 10 and place value may exist based on whether they had experienced Montessori mathematics instruction between the ages of 3 and 6 years.

### Arithmetic Accuracy and Decomposition Strategy

To be successful in more complex math problem solving, children must first learn to accurately and efficiently solve simple arithmetic problems in early elementary school (Cowan et al., 2011; Jordan, Kaplan, Oláh, & Locuniak, 2006). Children can arrive at solutions to addition problems through various strategies, each of which includes certain prerequisite skills. When asked to solve problems without paper and pencil, children typically use one of three types of addition strategies: (a) counting, (b) retrieval, and (c) decomposition (Geary, Bow-Thomas, Liu, & Siegler, 1996; Geary, Fan, & Bow-Thomas, 1992; Shrager &

Siegler, 1998). Counting involves enumerating both of the addends or counting up from one of the addends. Retrieval involves recalling the solution to a problem as a number fact stored in memory, rather than active computation. Decomposition involves transforming the original problem into two or more simpler problems, which often involves first solving for ten (e.g., base-10 decomposition: solving 6 + 5 by adding 6 and 4 to get to 10 and then adding 1 more).

The base-10 decomposition strategy is one of the most efficient mental strategies for accurately solving arithmetic problems, particularly when problems involve double-digit numbers (Ashcraft & Stazyk, 1981; Torbeyns, Verschaffel, & Ghesquière, 2004). Children who use this strategy tend to have a better understanding of the base-10 structure of the number system than those who do not use it (Laski et al., 2014). In fact, children and adults who frequently use decomposition to solve arithmetic problems tend to have higher math performance and overall math achievement scores than those who depend on counting strategies (Carr & Alexeev, 2011; Carr, Steiner, Kyser, & Biddlecomb, 2008; Geary, Hoard, Byrd-Craven, & DeSoto, 2004; Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). Thus, examining the frequency with which children use decomposition to solve arithmetic problems provides insight into their overall mathematics knowledge.

## Study Hypothesis and Research Questions

The present study was based on the hypothesis that the Montessori approach may help children to acquire base-10 and place-value understanding, as well as greater arithmetic accuracy, and to use a base-10 decomposition strategy to a greater extent than other traditional, non-Montessori programs. This hypothesis was based on the extent to which Montessori mathematics materials emphasize the base-10 structure of numbers and that children have opportunities to engage with these materials in the pre-primary program (ages 3 to 5 years).

Research indicates that concrete materials can support young children's mathematics learning but that not all materials are equally effective (e.g., Laski & Siegler, 2014; Siegler & Ramani, 2009; Uttal, O'Doherty, Newland, Hand, & DeLoache, 2009). A recent literature review identified four principles that make it more likely concrete materials will be effective for learning: (a) consistent use of manipulatives, (b) introduction of concrete representations of concepts before gradual progression to more abstract representations, (c) avoidance of manipulatives that represent everyday objects, and (d) clear explanation of the relation between the manipulative and the concept it represents (Laski et al., 2015). Further, this paper proposed that the Montessori materials used for teaching number concepts and the base-10 structure follow these principles. For example, the Montessori curriculum uses a small set of materials (e.g., the golden beads) consistently for several years of instruction, beginning with concrete representations of 10 bars and unit beads, and proceeding to more abstract representations using tiles with numerals. The materials used in Montessori instruction for mathematics also have an explicit and consistent system for representing place value through color coding (Laski et al., 2015; Lillard, 2005).

In addition to the quality of materials used in Montessori mathematics instruction, its emphasis on trading in addition also suggested it would engender a stronger understanding of base 10 and arithmetic in early childhood than other programs do. Evidence indicates that explicit instruction on how to use base-10 decomposition strategies for arithmetic is critical for learning how to accurately execute this strategy and for improving understanding of base 10 (Fuson & Li, 2009; Perry, 2000). Montessori math lessons emphasize the trading of units and tens as the preferred approach to multidigit arithmetic, starting with children's very first exposure to these kinds of problems (Montessori & Simmonds, 1917). In contrast, an analysis of the lessons included in typical non-Montessori curricula (e.g., TERC mathematics) found less emphasis on this approach than in the Montessori math program, particularly in kindergarten.

Based on our hypothesis that the Montessori approach may help children acquire base-10 and place-value understanding, as well as greater arithmetic accuracy, and to use base-10 decomposition strategy to a greater extent than other traditional, non-Montessori programs, we tested three specific research questions.

First, in early childhood (kindergarten and first grade), do students from Montessori and non-Montessori schools exhibit differences in (a) use of base-10 materials to represent number, (b) accurate solution of arithmetic problems, (c) strategy choice when solving arithmetic problems, and (d) accurate execution of decomposition strategies?

Second, do the differences in Montessori and non-Montessori approaches to mathematics persist and/or emerge later in more complex problem-solving? Specifically, do students from Montessori and non-Montessori elementary schools exhibit differences in understanding place value, in accurately solving arithmetic problems, and in strategy choice when solving arithmetic problems?

Third, do early differences in understanding of base 10 predict later differences in accuracy on arithmetic problems and place-value representations?

We predicted that, particularly at the end of the three-year cycle (kindergarten and third grade), children from Montessori schools would perform above their same-aged peers on tasks requiring conceptual understanding of base 10 and place value. Additionally, we predicted that children with greater base-10 knowledge would also be more likely to use decomposition strategies when solving addition problems.

## Method

### Participants

The study included a large group of kindergartners and first graders ($N = 150$) from Montessori and non-Montessori schools in a northeastern city. As shown in Table 1, 77 kindergartners (Montessori: $n = 48$; non-Montessori: $n = 29$. $M_{age} = 6$ years, 2 months) and 73 first graders (Montessori: $n = 56$; non-Montessori: $n = 17$. $M_{age} = 7$ years, 2 months) were included at Time 1 (T1). Approximately 30% of these children were tested again two years later at Time 2 (T2). Two cohorts of children participated at both T1 and T2. One cohort (Montessori: $n = 15$; non-Montessori: $n = 8$) was assessed at T1 as kindergartners ($M_{age} = 6$ years, 2 months) and again at T2 as second graders ($M_{age} = 8$ years, 5 months). The second cohort (Montessori: $n = 17$; non-Montessori: $n = 13$) was assessed at T1 as first graders ($M_{age} = 7$ years, 1 month) and again at T2 as third graders ($M_{age} = 9$ years, 4 months). There were no significant differences between the ages of children in the two programs at any grade level.

Table 1

*Descriptive Information of Participants Included at Time 1 (T1) and Time 2 (T2)*

| | T1 | | T2 | |
|---|---|---|---|---|
| | Kindergarten | First Grade | Second Grade | Third Grade |
| Montessori *n* | 48 | 56 | 15 | 17 |
| Non-Montessori *n* | 29 | 17 | 8 | 13 |
| Mean Age (months) | 74 | 86 | 101 | 112 |

### Procedure

**Time 1.** At Time 1 (T1), when children were in kindergarten or first grade, two tasks were administered over two sessions: a base-10 block task and an addition task.

In the base-10 block task, an experimenter presented children with unit blocks and 10 blocks and explained that the blocks could be used to show numbers. The experimenter took 10 unit blocks from the tray and lined them up against a 10 block while counting from 1 to 10 to demonstrate that one long block was the same as 10 small blocks. After being introduced to the task with two practice trials, each child was given five test trials. On each test trial, the experimenter presented a child with a different number card and asked the child to represent the number using blocks. The five trials included a random presentation to a child of the numbers 12, 16, 28, 34, and 61. The experimenter recorded how many unit blocks and 10 blocks the child used to represent each number and made notes about the child's response. For each trial, the experimenter coded whether the child (a) used only unit blocks, (b) used a canonical base-10 representation, which involved using the largest possible number of 10 blocks to represent tens and unit blocks to represent ones (e.g., showing 23 with two 10 blocks and three unit blocks), (c) used a noncanonical base-10 representation, which involved some base-10 blocks but not the maximum number, as well as unit blocks (e.g., showing 23 with one 10 block and thirteen unit blocks), or (d) none of the above.

In the addition task, children were presented with a series of individual addition problems, each problem printed on a separate piece of paper. The experimenter read each problem aloud and then gave children as much time as needed to solve the problem. Children were not provided with any supplies, such as paper or pencil, but were permitted to use their fingers or count aloud. The experimenter observed the child and recorded any overt signs of strategy use (e.g., if the child counted aloud, the tester noted use of a counting strategy). When there were no overt behaviors, the tester asked the participant how he or she figured it out after an answer was provided. Each problem was scored for accuracy. In addition, the experimenter coded the strategies children used as one of five categories: *count all*, *count-on*, *decomposition*, *retrieval*, and *other*. The count-all strategy was used when a child counted out each addend and then counted the total (e.g., to solve 5 + 3, a child would first count to 5, then count to 3, then finally count from 1 to 8). The count-on strategy was used when a child counted up from one addend the value of the second addend (e.g., to solve 5 + 3, a child would count from 6 to 8). Decomposition was used when a child transformed the original problem into two or more simpler problems, using either a previously memorized number fact or the base-10 properties of the number system (e.g., to solve 7 + 6, a child might first add 7 + 3 to get 10 and then add 3 more to arrive at 13). Retrieval was used when a child recalled the solution from memory. If a child guessed or used a strategy that could not be coded into one of the previous categories, the strategy was coded as other.

**Time 2.** At Time 2 (T2), when they were in second or third grade, children completed fifteen problems: five place-value problems (e.g., "Circle the largest number: 10101, 1901, 93001, 1899.") and ten arithmetic problems. In the set of arithmetic problems, five double-digit and mixed-digit addition and subtraction problems were contextualized within a story (e.g., "A grocery store had 89 bananas. It sold 27 bananas on Monday and 34 bananas on Tuesday. How many bananas were left in the grocery store on Wednesday?"). The remaining five arithmetic problems were decontextualized, where children were presented with double-digit addition, subtraction, and missing-term problems (e.g., 42 – 29 = ?) using only numerical symbols. Children were permitted to solve the problems either mentally or with the paper and pencil provided. For each problem, experimenters followed the same procedure used at T1 to evaluate children's accuracy and strategy use, with two differences in procedure. The first difference was to not code retrieval. The problems used double-digit numbers, and it is believed that children are able to recall answers only on simple problems (e.g., Geary et al., 2004). The second difference was the use of the code *written algorithm* when children wrote or described the written algorithm (e.g., "I lined up the numbers in my head and carried the one….").

## Results

### Research Question 1

To answer our first research question, we examined whether there were program and grade-level differences in accuracy on the base-10 block task and arithmetic, as well as strategy choice and accuracy when using a decomposition strategy. For this research question, we were interested in examining only the data from T1, so we used the entire sample ($N$ = 183).

**Base-10 Knowledge**. We ran a 2 (grade: kindergarten vs. first) × 2 (program: Montessori vs. non-Montessori) ANOVA on the percentage of block-task problems for which children correctly used a canonical representation of base 10. We found a main effect for grade, $F(1, 178) = 39.47$, $p < .001$, $\eta_p{}^2 = .18$ and a grade-by-program type interaction, $F(1, 178) = 14.32$, $p < .001$, $\eta_p{}^2 = .07$. Figure 1 presents the children's average accuracy, separated by grade and program type. First graders used accurate canonical representations on a greater percentage of trials than kindergartners, 90% ($SD$ = 28) versus 56%, ($SD$ = 43), respectively, $p < .001$. This effect varied by program: first graders who attended non-Montessori schools were more likely to use canonical representations than their kindergarten counterparts, $p < .001$, but there were no grade-level differences between both kindergartners and first graders attending Montessori programs. In other words, between kindergarten and first grade, children in non-Montessori programs demonstrated a substantial increase in their use of canonical representations (44% [$SD$ = 43] to 94% [$SD$ = 28] of trials), whereas Montessori children did not. This finding may be attributable in part to Montessori children having less room for improvement than non-Montessori children. In kindergarten, children from Montessori schools used correct canonical representations on 28% more problems compared to children from public schools. In sum, children who attended Montessori programs demonstrated an advantage in base-10 understanding in kindergarten relative to their non-Montessori peers, but non-Montessori children improved by the end of first grade such that there was no longer a difference between programs.

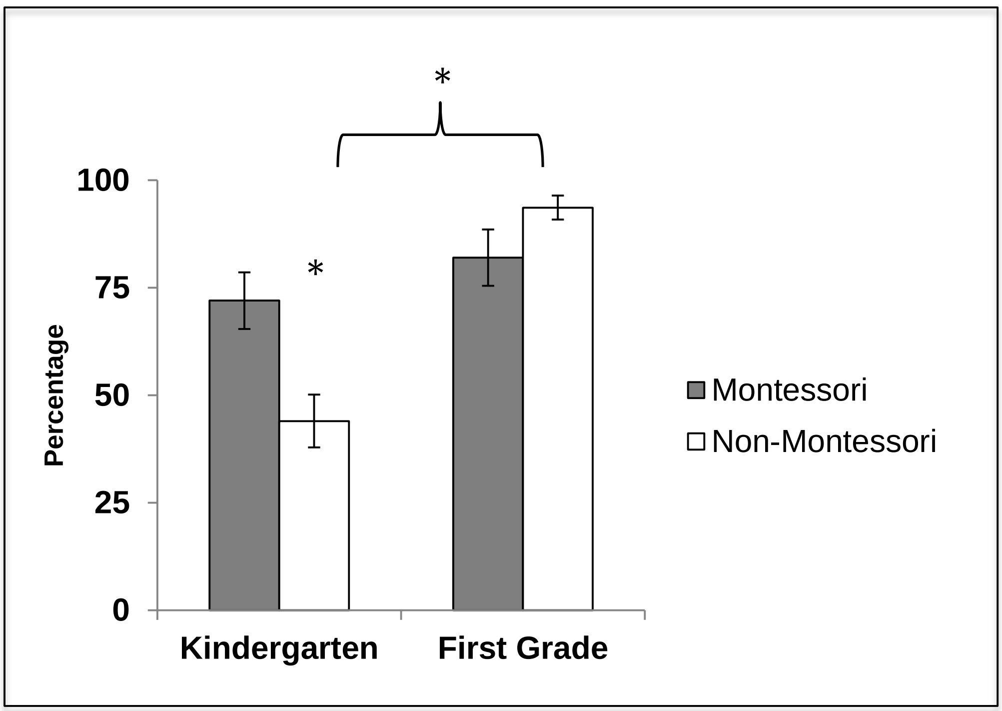


*Figure 1.* Percentage of correct base-10 canonical representations of base 10 at T1. Error bars represent standard error.
* $p < .001$.

**Arithmetic Accuracy.** A 2 (grade: kindergarten vs. first) × 2 (Program: Montessori vs. non-Montessori) ANOVA on the percentage of problems answered correctly found a main effect for grade, $F(1, 178) = 43.62$, $p < .001$, $\eta_p{}^2 = .20$. First graders correctly answered more addition problems than did kindergartners, $p < .001$, 87% vs. 63%.

**Arithmetic Strategy.** A 2 (grade: kindergarten vs. first) × 2 (program: Montessori vs. non-Montessori) MANOVA on the percentage of problems for which children used a *counting*, *decomposition*, *retrieval*, or *other* strategy found a multivariate main effect for grade, $F(4, 175) = 16.57$, $p < .001$, $\eta_p{}^2 = .27$, such that the distribution of strategies used by children changed between kindergarten and first grade. To better understand the multivariate effect, we examined the result of the univariate analyses and found main effects for grade for each strategy type. Kindergartners used counting, $F(1, 178) = 25.98$, $p < .001$, $\eta_p{}^2 = .13$, and other, $F(1, 178) = 6.30$, $p = .01$, $\eta_p{}^2 = .03$, more frequently than first graders; counting and other were used on 65% ($SD = 31$) and 11% ($SD = 20$) of addition problems by kindergartners but on 39% ($SD = 31$) and 5% ($SD = 8$) percent by first graders. In contrast, first graders used decomposition, $F(1, 178) = 38.03$, $p < .001$, $\eta_p{}^2 = .18$, and retrieval, $F(1, 178) = 45.59$, $p < .001$, $\eta_p{}^2 = .20$, more frequently than kindergartners; decomposition and retrieval were used on 10% ($SD = 18$) and 9% ($SD = 11$) of addition problems by kindergartners but on 32% ($SD = 25$) and 23% ($SD = 14$) by first graders. There was no main effect for program or program-by-grade interaction. Thus, first graders were more likely to use sophisticated strategies than kindergartners, regardless of program, and at each grade Montessori and non-Montessori students used similar strategies to solve the addition problems. Figure 2 presents the average percentage of trials for which kindergartners and first graders used each type of strategy.

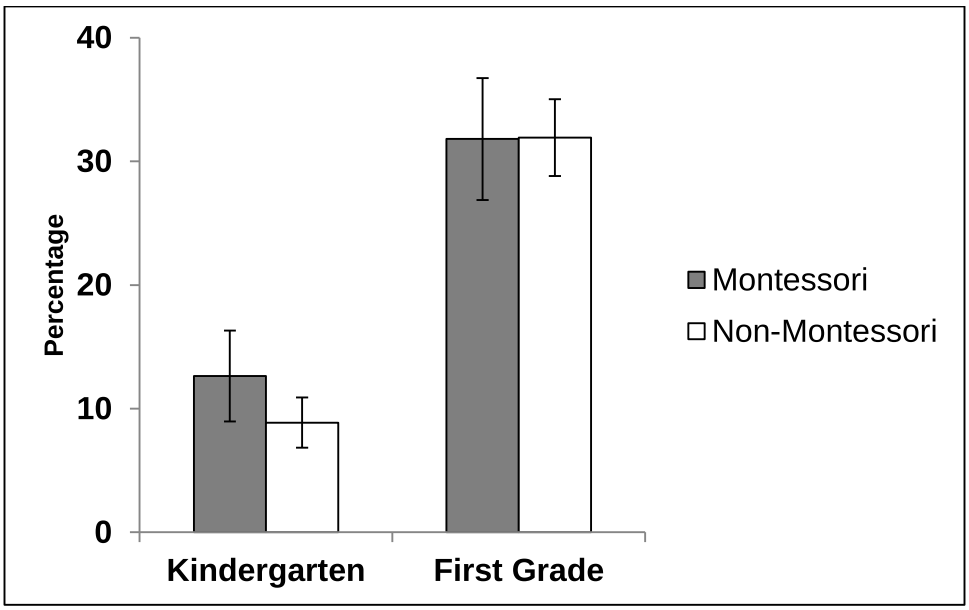


*Figure 2.* Percentage of problems on which children used a decomposition strategy at T1.

**Decomposition Accuracy.** A 2 (grade: kindergarten vs. first) × 2 (program: Montessori vs. non-Montessori) ANOVA on the percentage of problems for which children correctly used a decomposition strategy found a main effect for grade, $F(1, 175) = 36.10$, $p < .001$, $\eta_p{}^2 = .17$, but no main effect for program or grade-by-program interaction. First graders used a decomposition strategy correctly on 32% ($SD = 25$) of the problems on which they attempted it, whereas kindergartners executed the strategy correctly on only 11% ($SD = 19$) of the problems on which they attempted it.

### Research Question 2

To answer our second research questions about whether children who have experienced Montessori approaches to mathematics through primary school demonstrate later advantages, we examined whether there were program and grade-level differences in accuracy on the place-value and arithmetic problems at second and third grades.

**Place-Value Knowledge**. A 2 (grade: kindergarten vs. first) × 2 (program type: Montessori vs. non-Montessori) ANOVA on accuracy on place-value problems found no main effects for grade or problem type, and there was no interaction between grade and program..

**Arithmetic Accuracy.** Preliminary analyses revealed no differences in children's accuracy on contextualized and decontextualized problems; thus, these two categories were combined to form an overall arithmetic measure. A 2 (grade: second vs. third) × 2 (program type: Montessori vs. non-Montessori) on the percentage of arithmetic problems answered correctly found a main effect for grade, $F(1, 49) = 5.66$, $p = .02$, $\eta_p^2 = .10$. Second graders accurately answered 64% ($SD = 32$) of problems, whereas third graders accurately answered 81% ($SD = 19$) of problems. There was no main effect for program and no interaction between program and grade.

**Arithmetic Strategy.** Finally, we examined grade and program differences on the percentage of arithmetic problems on which children used a counting, decomposition, written algorithm, or other strategy. A 2 (grade: second vs. third) × 2 (program type: Montessori vs. non-Montessori) MANOVA found a multivariate effect of grade, $F(4, 46) = 5.68$, $p = .001$, $\eta_p^2 = .33$ and a trend for a grade-by-program interaction, $F(3, 46) = 2.50$, $p = .06$, $\eta_p^2 = .18$.

To better understand the multivariate effect, we examined the result of the univariate analyses and found a main effect of grade in the frequency with which children used a counting strategy, $F(1, 49) = 10.37$, $p = .002$, $\eta_p^2 = .18$. Children used counting on 14% ($SD = 20$) of problems in second grade and 2% ($SD = 6$) of problems in third grade. We found a grade-by-program interaction for use of a written algorithm strategy. Children in Montessori schools used written algorithm on about 46% of problems in both second and third grades. However, children's percentages in non-Montessori schools increased from using written algorithm on 25% ($SD = 29$) of problems in second grade to 77% ($SD = 28$) of problems in third grade. Figure 3 presents the percentage of problems on which children chose to use a written algorithm strategy by program type and grade. When looking at the percentage of problems on which children were coded as using other strategies, there was both a main effect for grade, $F(1, 49) = 10.14$, $p = .003$, $\eta_p^2 = .17$, and a grade-by-program interaction, $F(1, 49) = 4.2$, $p = .046$, $\eta_p^2 = .08$. Overall, children used other strategies on 6% ($SD = 14$) of problems in second grade and on 0.7% ($SD = 4$) of problems in third grade. Children's use of other strategies in non-Montessori programs decreased by 10%, while the use of other strategies by children in Montessori programs decreased by less than 3%.

### Research Question 3

To examine whether early differences in performance might be related to later ones, we ran a series of correlational analyses between children's performance on assessments at T1 and their performance on assessments at T2. In particular, we were interested in whether base-10 knowledge in kindergarten and first grade predicted arithmetic performance in second and third grades and whether this relation varied by program type. These analyses included only the subgroup of children who participated in both the T1 and T2 studies.

**Cohort 1.** As shown in Table 2, for cohort 1 we found that the percentage of trials in which kindergartners used base-10 canonical representations of number was positively correlated with accuracy on arithmetic problems in second grade among children from non-Montessori programs and for accuracy on arithmetic and place-value problems in second grade among children in Montessori programs. In addition, accuracy on addition problems in kindergarten was positively correlated with accuracy on arithmetic problems in second grade among children in non-Montessori programs. However, accuracy on addition problems at T1 was not correlated with any T2 measures among children from Montessori programs. Finally, the percentage of trials in which children used a decomposition strategy in kindergarten was not correlated with accuracy on any type of arithmetic problem in second grade among children from non-Montessori programs or Montessori programs.

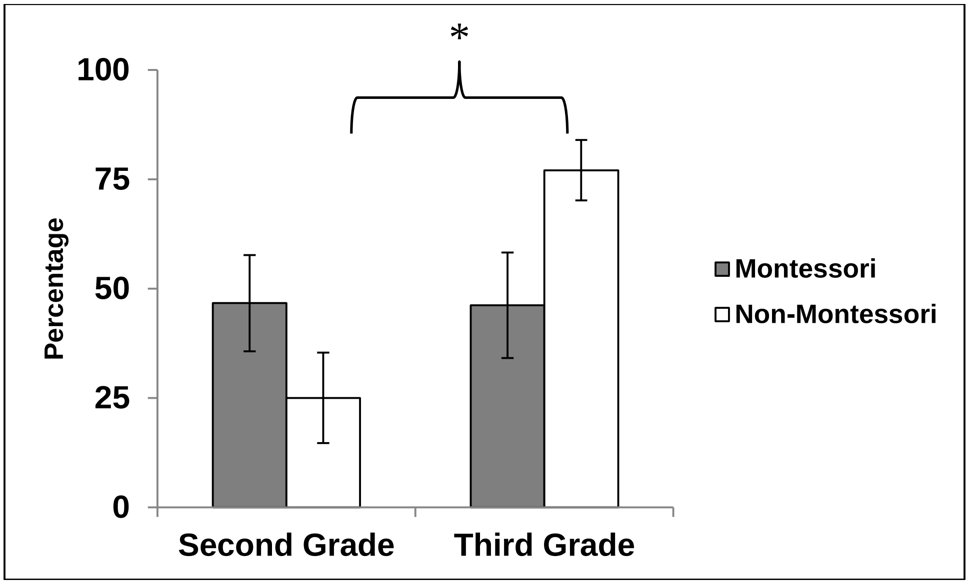


*Figure 3.* Percentage of problems on which children used a written algorithm strategy at T2.

**Cohort 2.** We ran identical correlation analyses for children from cohort 2. As Table 2 shows, the percentage of trials in which first graders used base-10 canonical representation of numbers as well as accuracy on the T1 arithmetic assessment were both correlated with accuracy on place-value problems among third graders in non-Montessori programs. However, base-10 canonical representation and T1 arithmetic accuracy were not correlated with any T2 measures among third graders from Montessori programs. The percentage of trials in which first graders used a decomposition strategy was not correlated with any third-grade measures among children from non-Montessori programs but was correlated with accuracy on arithmetic problems and place-value problems in third grade among children from Montessori programs.

Table 2

*Correlation Coefficients (Pearson's* r*) Between Accuracy Percentages on Assessments from Time 1 (T1) and Time 2 (T2)*

**Kindergartners and second graders from non-Montessori programs**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1. T1 Base-10 Canonical Representation | - | | | | |
| 2. T1 Arithmetic | 0.924** | - | | | |
| 3. T1 Decomposition | 0.592 | 0.478 | - | | |
| 4. T2 Arithmetic | 0.948** | 0.799* | 0.403 | - | |
| 5. T2 Place Value | 0.552 | 0.522 | 0.622 | 0.713* | - |
| 6. T2 Decomposition | 0.587 | 0.473 | 0.408 | 0.455 | –0.032 |

**Kindergartners and second graders from Montessori programs**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1. T1 Base-10 Canonical Representation | - | | | | |
| 2. T1 Arithmetic | 0.458 | - | | | |
| 3. T1 Decomposition | 0.352 | 0.451 | - | | |
| 4. T2 Arithmetic | 0.600* | 0.426 | 0.506 | - | |
| 5. T2 Place Value | 0.725** | 0.394 | 0.491 | 0.834** | - |
| 6. T2 Decomposition | 0.317 | 0.195 | –0.073 | 0.194 | 0.29 |

**First and third graders from non-Montessori programs**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1. T1 Base-10 Canonical Representation | - | | | | |
| 2. T1 Arithmetic | 0.458 | - | | | |
| 3. T1 Decomposition | 0.419 | 0.686** | - | | |
| 4. T2 Arithmetic | 0.224 | 0.317 | 0.112 | - | |
| 5. T2 Place Value | 0.560* | 0.505* | 0.406 | 0.491* | - |
| 6. T2 Decomposition | 0.286 | 0.34 | 0.444 | 0.121 | 0.416 |

**First and third graders from Montessori programs**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1. T1 Base-10 Canonical Representation | - | | | | |
| 2. T1 Arithmetic | 0.204 | - | | | |
| 3. T1 Decomposition | –0.041 | 0.732** | - | | |
| 4. T2 Arithmetic | 0.220 | 0.409 | 0.577* | - | |
| 5. T2 Place Value | 0.03 | 0.302 | 0.664* | 0.718* | - |
| 6. T2 Decomposition | 0.253 | 0.284 | 0.316 | –0.417 | –0.005 |

*Note:* T1 represents tasks from Time 1; T2 represents tasks from Time 2. T1 and T2 decomposition refer to the percentages of trials in which children used a decomposition strategy, regardless of whether they accurately answered the problem.

* $p < .05$ level, two-tailed

** $p < .01$ level, two-tailed

## Discussion

This study tested the hypothesis that the Montessori approach promotes better base-10 and arithmetic understanding than other traditional programs, given the emphasis of these concepts in Montessori materials and instruction. Overall, the study found no evidence in support of this hypothesis. By first grade and through the end of third grade, no differences were found in the mathematics knowledge of children from Montessori and non-Montessori programs. In the discussion that follows, we offer potential explanations for and implications of the findings.

### Early but Not Later Differences in Base-10 Understanding

The results of this study revealed that children in Montessori programs showed an advantage in base-10 understanding in kindergarten, as compared with peers from more traditional schools. However, Montessori students did not show the same advantage in first, second, and third grades. There are several possible explanations for this finding. First, the specific group of Montessori kindergartners who participated in this study might have been particularly advanced. This explanation seems unlikely, however, because the same children were tested in second grade and showed no advantage at that time. A more plausible explanation is that Montessori might have an especially strong preschool program or that more of the Montessori kindergartners had experienced preschool instruction, given that kindergarten is the final year in the three-year cycle, giving children in kindergarten an advantage over their peers from other programs. However, by first grade, non-Montessori peers might have had enough time to catch up.

The pattern of correlations between base-10 understanding at T1 and place-value knowledge at T2 support this "catch-up" explanation. The place-value assessment given to children in second and third grades was designed to be an age-appropriate test analogous to the base-10 assessment used in kindergarten and first grade. We predicted children who used canonical base-10 representations in kindergarten and first grade would perform more accurately on place-value problems two years later. We found that the percentage of canonical base-10 representations in kindergarten was correlated with place-value performance, but not other skills, in second grade among children from Montessori schools; the percentage of canonical representations of base-10 in first grade was positively correlated with place-value accuracy, but not other outcomes, for third graders from non-Montessori schools. This different pattern of correlations supports the view that Montessori children acquired these concepts primarily in kindergarten, whereas non-Montessori students did so in first grade. Importantly, second graders from both schools performed equally as accurately on the place-value assessment, suggesting that timing of acquisition had little-to-no effect on later performance.

Another explanation, not mutually exclusive, may be that the Montessori approach focuses on a wider range of math concepts than typical non-Montessori instruction during this time period, in other words, children receive differential practice in base 10 across programs. Further research with a larger number of students is necessary to gain insight into why this Montessori advantage existed in kindergarten but not in elementary school. It would also be worthwhile to examine differences on broader measures of mathematics achievement.

### Longitudinal Patterns of Arithmetic Accuracy and Strategies

While there were no differences in arithmetic accuracy between children from Montessori and non-Montessori schools at either T1 or T2, children from the different programs did exhibit different developmental trajectories. Arithmetic accuracy in kindergarten appeared to predict arithmetic accuracy in second grade for children from non-Montessori schools but not for children from Montessori schools. This finding suggests that, in more traditional school settings, teaching practices and curricula might require children to build upon past knowledge when learning to solve more advanced problems. By the middle of elementary school, children who did not develop basic arithmetic understanding at the start of schooling might have a challenging time accurately solving more advanced problems. Likewise, those children who were particularly advanced in kindergarten might be able to use their early knowledge to continue to succeed. However, results from this study suggest that early arithmetic ability is not predictive of advanced problem-solving skills for children in Montessori programs. Montessori teaching practices might encourage children to draw from a range of skills other than arithmetic ability to solve more advanced arithmetic in middle elementary school or might consistently review arithmetic skills over the three-year period, making kindergarten skill-level less influential.

Despite not exhibiting differences in arithmetic accuracy, children in Montessori and non-Montessori programs executed different strategies to obtain their answers in second and third grades. Strategy use did not differ by program type in kindergarten and first grade. In both second and third grades, children in Montessori schools showed a fairly even split between using written algorithms and decomposition strategies. However, in non-Montessori schools, children shifted from using a combination of written algorithm, decomposition, and counting strategies in second grade to using a written algorithm strategy on approximately three quarters of problems in third grade. These results suggest that Montessori curriculum may emphasize the use of algorithms to solve problems less than non-Montessori schools do. This difference may be because Montessori programs continue to use concrete materials throughout the early elementary school years more than non-Montessori programs. Importantly, there seemed to be no disadvantage in the shift toward written algorithms for non-Montessori children: these children demonstrated accuracy and place-value knowledge comparable to that of Montessori children. Further research is necessary to understand whether the strategies children use to execute arithmetic problems in third grade are predictive of math outcomes later in elementary school.

**Implications for Montessori Education**

In sum, the results from this longitudinal study indicate that the Montessori approach may offer an early advantage over non-Montessori programs in helping children understand critical math concepts, but this gain does not translate into a long-term advantage. The findings raise at least two questions for Montessori educators to consider.

First, as children transition to elementary programs, what can be done to maintain and build on the advantage kindergartners demonstrate in base-10 understanding? Children from Montessori schools did not demonstrate improvement on base-10 understanding between kindergarten and first grade, despite not being at ceiling in kindergarten. In contrast, children in non-Montessori programs demonstrated substantial improvement between kindergarten and first grade. Further, it is important to note that the advantage did not re-emerge at the end of the three-year cycle: no difference remained in place-value understanding between Montessori and non-Montessori children in third grade. A better understanding of what happens when children transition from the Children's Garden to the elementary program is needed. There may be unnecessary repetition in lessons; alternatively, the transition to abstract representations could occur more rapidly.

Second, how can instruction help children generalize and transfer their understanding of bead bars and units to arithmetic tasks and strategies? Previous research has demonstrated that kindergartners' representation of base 10 contributes to the frequency with which they attempt to solve arithmetic problems with base-10 decomposition (Laski et al., 2014). However, in the current study, decomposition at T1and T2 were not correlated with base 10 or place value for any group of children in this study.

There is increasing evidence that children require explicit guidance and instruction to abstract concepts from concrete materials or to see connections between two concepts (Carbonneau, Marley, & Selig, 2013; DeLoache, Peralta de Mendoza, & Anderson, 1999; Laski & Siegler, 2014). According to the cognitive-alignment framework, a theoretical framework for instructional design, even if the concrete materials are ideally designed, learning is unlikely to occur if procedures and didactic statements do not direct children's attention to the relevant features (Laski & Siegler, 2014). Thus, educators should consider how to explicitly show children that their base-10 knowledge is beneficial in the use of decomposition for mentally solving addition problems.

## Conclusion

Children educated with Montessori curricula or with more mainstream curricula likely receive very different instruction when learning foundational math concepts. This study reveals similar levels of accuracy in arithmetic and place value for both Montessori and non-Montessori students. These results demonstrate that there are many different, effective ways to approach early math education. Future research spanning a longer time frame and more complex concepts might shed greater light on whether there are lasting effects of different educational approaches.

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