Teaching in the Montessori Classroom: Investigating Variation Theory and Embodiment as a Foundation of Teachers' Development

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Abstract: The theory of Montessori education has been interpreted by some researchers to be vaguely formulated. However, as shown in previous research, Maria Montessori's didactic approach to teaching and learning mathematics is fully consistent with variation theory and the theory of embodiment. Dr. Montessori used the theoretical concept of *isolation of quality*, which means that the learning objects have to be kept identical except for one variable, which has to differ to be perceptible. This concept is in alignment with variation theory, which emphasizes variation as a necessary condition for learners to discern aspects of an object of learning. The other theory applied in this article is the theory of embodiment: important cognitive functions are fundamentally grounded in action that is concordant with Dr. Montessori's view that mind and movement are parts of the same entity.

This article reports on a qualitative single-case study with a formative intention in which we investigated the significance of being acquainted with variation theory and the theory of embodiment when working with Montessori material. The study analyzes a teacher's mathematics presentations with the Montessori material and the children's work with this material, using Epistemological Move Analysis, which focuses on how the teacher directs children's learning. The analysis was shared with the teacher to support her awareness of the ways teaching can be developed from a variation and embodiment theoretical perspective. Results show that the teacher's awareness of why a specific learning object must be treated in accordance with variation theory and embodiment seems to promote a more constructive and effective way to direct children's learning.

Maria Montessori described in her literature (e.g., Montessori, 1912/1964; Montessori, 1914/1965) how various didactic materials should be presented in Montessori education. However, as some interpreters of the pedagogy have pointed out (e.g., Feez, 2007; Lillard, 2005; M. M. Montessori, 1992), her description of the theory is vague. As Ahlquist and Gynther (2019) noted, Dr. Montessori gave detailed instructions on how to present the material, but she was not as explicit about the underlying didactic motives for why it should be done in the manner she described.

Cossentino (2009) stated that the focus on how Montessori material is managed in Montessori teacher training leads to the intention of Montessori pedagogy. In the present article, we take a closer look at Dr. Montessori's pedagogical intention and argue that teachers need to understand why teaching has to be facilitated and structured in accordance with the didactic intention. Such understanding is crucial in creating favorable conditions for children's learning, as well as improvement in teaching. For this to happen, teachers' lessons should be grounded in a didactic, theoretical understanding.

One key principle, which some interpreters recently have noted at a more theoretical level (e.g., Marton, 2006; Marton, 2015; Marton & Signert, 2008; Signert, 2012), is the use of variation and invariance in the training of the senses as practiced in Montessori preschools . As Marton (2015) pointed out, this training of the senses is carried out in accordance with what is emphasized in variation theory. However, Ahlquist and Gynther (2019) showed that variation theory is also valid for areas other than sensorial training. Ahlquist and Gynther showed that the use of Montessori mathematics material plays an essential role in identifying different aspects of the learning object that, according to variation theory, are crucial for a learning outcome, and the use is fully consistent with variation theory. As in Montessori education, where the use of the body is central (Montessori, 1912/1964; 1914/1965; 1948/1972; 1949/1982), Ahlquist and Gynther also stressed the importance of body-based investigations for understanding mathematical concepts. The use of variation and invariance, as well as the awareness of the theory of embodiment, can therefore be seen as a key principle in Montessori education in general, not just in sensorial training, and consequently functions as part of "a platform for teachers and others when reviewing how different topics are treated in various Montessori environments" (Ahlquist & Gynther, 2019, p. 9).

This platform was the starting point for our study, initiated in spring 2019, which analyzed a teacher's presentations in mathematics with the Montessori material and children's work with this material. The analyzed lessons were then shared with the teacher to support her awareness of the ways teaching can be developed from a variation and embodiment theoretical perspective. The aim of the study was to investigate the meaning of teachers being familiar with the underlying theories of why the material should be presented in the way Dr. Montessori described it. We established two research questions for the study:

1. What can we distinguish as an important result of the intervention between the teacher and the researchers in explicitly connecting Montessori lessons to variation theory and embodiment?
2. In what ways does the children's work change character after receiving lessons informed by variation theory and embodiment?

Montessori Education, Variation Theory, and Embodiment

In her book *Psychogeometry* (2011), Dr. Montessori considered the challenges in teaching geometry and arithmetic. She did not agree with the idea that the only thing that matters is that teachers should start with the concrete and move on to the abstract by beginning with what is easy and then, little by little, moving on to more-advanced studies. It is not just finding the most logical way to teach that will solve the problem of teaching mathematics. What is important is "that the pupil agrees to receive the knowledge and is able to pay attention or, in other words, is interested" (Montessori, 2011, p. 4). Therefore, it is essential to find the necessary conditions for "unfolding" or developing "the art of allowing joy and enthusiasm" (Montessori, 2011, p. 5). In the same chapter, Dr. Montessori broadened the challenges in teaching by discussing the concept of understanding: How can understanding become something active and not just storing a number of understood entities without any connection to interest? Here, Dr. Montessori was very clear when she pointed out the difference between a human being and a machine. To learn something demands effort, but it is not possible to require effort when there is a lack of interest; on the other hand, when a person is interested, he or she is generally willing to put a lot of effort into the work. To become interested, a child must have the opportunity to make discoveries; at the same time, however, it is not possible to create a theorem without the proper mathematical language.

The geometry material, according to Dr. Montessori (2011), is designed to attract a child's interest in a way that teachers cannot, because the child's mind is not mature enough to receive explanations without having done his or her own explorations. The geometry material is constructed to discover the relationship between different shapes; handling the material allows the eye and mind to perceive the state of things, which enables the child to reveal what distinguishes one figure from another.

> *In other words, if we realize that there are abstract and quibbling reasonings on things that are complicated, but the things themselves, when materially observed, are much simpler, it becomes immediately evident how an alternative path can be opened up for the elementary study of geometry, leading to unforeseen results.* (Montessori, 2011, p. 56)

When viewing teaching from a variation theoretical perspective, it follows that the aim of teaching is to create conditions that will help the learner perceive the necessary aspects of the object of learning and the relationships between them. Learning, according to Marton and Booth (1997) and Marton (2015), is when the learner has learned some aspects that he or she was not aware of before. Variation theory, therefore, as well as Montessori education, stress that the relationship between what can be seen as the whole and its parts must be perceived by the learner if learning is to take place. Lo (2012) argued:

> *There must be a whole to which the parts belong before the parts can make sense to us. We cannot learn more details without knowing what they are details of. When the whole does not exist, learning will not be successful.* (p. 26)

Dr. Montessori made the same point: "To teach details is to bring confusion; to establish the relationship between things is to bring knowledge" (Montessori, 1948/1996, p. 58).

From a variation theoretical perspective, a learner has to be aware of the differences between at least two features to be able to discern them (Marton, 2015). For example, to discern the shape of a triangle, the learner has to be exposed not only to a triangle but also to other shapes (e.g., a square). In that way, learners will be able to discern a triangular shape at the same time that they discern what is not triangular. However, other aspects, like color and size, have to be kept invariant to make it likely that the learner discerns the aspect in focus (i.e., shape). As Dr. Montessori (1948/1972) wrote, "If, for example, we want to prepare objects to be used in distinguishing colors, we must make them of the same material, size, and dimensions, but then see that they are of different colors" (p. 101). Once learners have found the meaning by contrast, they have to generalize the aspect that had previously been separated. If the aspect, for instance, is shape, generalization is achieved by keeping the shape invariant and varying other aspects, such as color and size. From a variation theoretical perspective, it is important that such a generalization always be preceded by contrast (Marton, 2015). The final step is to let learners experience simultaneous variation in all relevant aspects. In variation theory, this pattern of variation is called *fusion*: "it defines the relation between two (or more) aspects by means of their simultaneous variation" (Marton, 2015, p. 51). In the case of a triangle, learners will experience, for instance, that any triangle that appears—whatever its size, color, length of sides, or different kinds of angles—is still a triangle.

Furthermore, learning, according to Dr. Montessori (1948/1972; 1949/1982), manifests itself through experiences in the environment; consequently, she considered bodily actions to be central in shaping our experiences and perceptions of the world around us. This view of learning is in accordance with the theory of embodiment, which sees meaning and cognition as deeply rooted in our physical existence. The embodied mind is not only an organ situated inside our body; according to Lakoff and Johnson (1999), it is also our bodily experience and interaction that supports our systems of thought. Embodiment is considered to be action and perception, grounded in the physical environment.

> *Meaning is embodied. It arises through embodied organism-environment interactions in which significant patterns are marked within the flow of experience. Meaning emerges as we engage the pervasive qualities of situations and note distinctions that make sense of our experience and carry it forward. The meaning of something is its connections to past, present, and future experiences, actual or possible.* (Johnson, 2007, p. 273)

Dr. Montessori (1949/1982) wrote that the hand explores and communicates with the brain at the same time the brain guides the hand. According to Dr. Montessori (1949/1982) this similarity supports the learning outcome since it is through these explorations that the mind not only has the power to imagine but can also assemble and reorganize its mental content. The similarity between the hand and the brain is consistent with Merleau-Ponty (2004), who stated that movement must be understood as an original intentionality and that consciousness does not mean "I think" but rather "I can" (p. 159). The use of Montessori material can be seen as an expression of this crucial standpoint.

Method

This study is a qualitative, single-case study that includes a formative intervention. *Formative intervention* means that analysis of the collected data and how the teaching can be developed from a variation theoretical perspective and awareness of embodiment were shared and discussed with the teacher throughout the study. The design of the study promoted active participation by both the researchers and the teacher regarding the implementation of the actions that occurred. The intention was to enrich the teacher's own learning of how to give presentations and prepare activities in mathematics.

The data were collected in field notes from several observations in a mixed-age Montessori class in a large city in Sweden, with 7- and 8-year-old children in their first and second school year. Before the study, a letter describing the study, including a guarantee that the school's location and the children's names would be anonymized, was sent to the parents with a form to be detached, signed, and returned to the principal; all parents gave their approval. The teacher informed the children about the study, and all children could choose not to participate in the study and to instead work with their classmates in another classroom; however, none chose this option. According to Stockholm University's Research Support Office, the study did not require approval from the Swedish Ethical Review Authority.

The subject teacher had received Montessori training from a Swedish university, was a licensed elementary teacher, and was chosen because of her specialized background in mathematics. We followed the teacher's presentations in mathematics through five sessions and observed the children's learning during a period of 3 weeks. The study includes sections in which the researcher interacted with the children during their activities to identify their learning ability and understanding. The researchers' observations focused on what the teacher or researcher said to the children and how the material was managed. Actions and expressions unrelated to the content being treated were disregarded. During the observed lessons and activities, the researchers took brief notes related to the focus of the study. Later that day, they added to these notes and developed them into more complete and detailed descriptions. To give a clear, explicit picture of the observations, and in alignment with Yin's (1994) recommendations, we decided to include long descriptions of the observed activities in the reported findings. For better access to classroom events, we decided not to use video or audio recording during observations. Both researchers were in the classroom at the same time, which helped reduce subjective understanding and increase the reliability of the data collected.

The teacher's role in the children's learning process, as well as the researchers' role as participants, was analyzed with the aid of Epistemological Move Analysis (EMA), which analyzes a teacher's role in students' learning process. (Lidar, Lundqvist, & Östman, 2006; Lundqvist, Almqvist, & Östman, 2012). In our analysis, we focused on how the teacher directed the children's learning in different ways by using what Lidar et al. (2006) referred to as *epistemological moves*, which we found suitable for the study. These different moves were (a) *instructional moves,* which instruct children and direct them how to act so they can see what is worth noticing (in our study, this meant how to use the Montessori material to comprehend and define the desired learning outcomes); (b) *confirming moves*, when teachers agree with (i.e., confirm) what children say or do by, for instance, giving positive feedback; (c) *reconstructing moves,* which are used when children pay attention to what they have noticed but have not yet comprehended, giving them the opportunity to reflect on their experiences in the work with the material; (d) *reorienting moves,* which encourage and challenge children to try out another way to deal with the task; and (e) *generative moves,* which enable children to generate understanding by reporting on the important knowledge they have perceived in the activity.

The collected data also included two interviews with the teacher in which we discussed the lessons and interactions with pupils. At the teacher's request, these interviews took place at Stockholm University and were recorded and later transcribed. The interviews were open-ended and semistructured; no interview guide was used, but the use of target questions provided insight into the teacher's thoughts about her teaching. After the interviews were transcribed, we followed up with the teacher for clarification and further details, covering several issues: the aim of the presentations; how the teacher planned to make complex concepts intelligible to the children; how she had planned the follow-up activities in mathematics for the children; and the roles of the teacher and researchers as participants in the children's learning process, as analyzed with the aid of EMA.

We also asked the teacher to reflect on how many children she chose for the presentation and how the environment was set to prepare for the activities in mathematics. The teacher gave her opinion on the lessons: what was successful and what she could have done in a different way. The discussion included our observations of the teacher's presentation and what we noticed during our interaction with the children, for example, how the children interpreted the activity, how they managed to complete the exercises, and what we noticed about their learning outcome. Discussing these sessions and learning activities with the teacher created conditions for development of the presentations. Together with the teacher, we agreed on ways to develop presentations and to set up new learning activities in which variation theory and the theory of embodiment were more evident. We maintained this work model throughout the study, so we were able to test how the children responded to the enhanced designs of the presentations.

Findings

In this section, we present patterns we identified while observing the teacher's presentations and the children's individual work. To illustrate how these patterns appear in the activities, long descriptions of observations, presented as narratives, are included.

Teachers' Presentations

In our initial observations of the teacher's presentations, we noticed that the feature they shared was that the children who participated in them mostly received information from the teacher (i.e., instructional moves), rather than opportunities to express how, or to what extent, they understood the content. Thus, in the first observed presentation, no reconstructing, reorienting, or generative moves were used by the teacher.

For example, in one instance the teacher had gathered 14 children for a presentation of polygons. The children sat on a circle-shaped carpet. A drawer[1] containing polygons was placed in front of the children, and the teacher began the presentation by saying, "Today, I am going to present polygons. *Poly* means *many* and *gon* means *corner*. What then does *polygon* mean?" One of the children quickly answered, "Many corners," which the teacher confirmed with a nod. The teacher then placed beside the drawer cards with the numbers 5, 6, 7, 8, 9, and 10. Next, the teacher picked up the pentagon and slowly, with a circular movement, felt along all the sides of the pentagon with her index and middle fingers as she counted the corners of the pentagon. Then she said, "*Gon* means *corner* and *five* in Greek is *pente*. So, what do you think is the name of this polygon?" One of the children immediately answered, "Pentagon." The teacher then presented the hexagon, heptagon, octagon, enneagon, and decagon in the same way. At the end of the presentation, the teacher told the children to feel the sides of each polygon with their index and middle fingers and then draw the polygons in their geometry books, writing the name of each polygon they had drawn.

[1] The drawer used in the presentation is part of the Geometric Cabinet, which was described in a previous study by Ahlquist and Gynther (2019).

At the end of the day of the presentation, we sat with the teacher and shared our reflections about what we had observed. The teacher told us, among other things, that she was not satisfied with the presentation of the polygons because she had not succeeded in engaging the children. According to the teacher, this result was also because the number of children present was quite large, making it even more challenging to engage all of them. As a result, we began to discuss what, in Montessori education, is a suitable number of children for a presentation like this; in our experience, Montessori teachers frequently discuss this issue. Dr. Montessori's own writings (e.g., Montessori, 1912/1964) referred to dialogues and discussions with groups of children, and we can assume the groups were of a manageable size. If dialogue and discussion are to take place and children are to have the time to explore, touch, and trace the material, there must not be too many children. A well-balanced number of children allows the teacher to follow each child's understanding of the material and its mathematical content. The meaning of a well-balanced number of children was shown by Blatchford (2003), who stated that "there is a strong suggestion that in a small class a teacher will more easily be able to provide at least some aspects of effective scaffolding for her pupils" (p. 590). The groups should not be too small either, as children are successful when they help each other by reasoning and explaining their own understanding (Wiliam, 2019).

We therefore agreed that in the next session we observed, the teacher should try to make a similar presentation to fewer children, letting them describe in more detail the similarities and differences between the geometrical shapes. We assumed that the children's knowledge would then be apparent to the teacher, thereby creating conditions for the teacher to direct the children's learning during the presentation with other epistemological moves besides instructional ones. At the same time, according to variation theory, the different shapes would be contrasted with others, making the aspect that defines each shape clearer for the children. We also discussed the probable critical concept for distinguishing different polygons: the corners of the shapes. However, when reviewing how the different polygons were presented, we all questioned whether it was wise for the teacher to trace the sides of the pentagon with her index and middle fingers while counting the corners. The teacher said this was what she had been taught to do in her Montessori training. We asked her if that might not confuse the children, as the teacher was supposed to count the corners and not the sides. After reflection, she agreed and said she wanted to change how she presented polygons the next time. We also encouraged her to reflect on the critical aspects of the learning object in the presentation to come and how to better engage the children.

In the next session, we observed her present quadrilaterals to the children. This time only five children were invited to participate, and a drawer[2] containing various quadrilaterals was placed in front of them. Before she directed children's attention toward the drawer, the teacher showed them the equilateral triangle she held in her hand. As it is not important to distinguish each individual child in this presentation, we refer to them as *child* in the following section.

> Teacher: "What can you tell me about this triangle?"
> Child: "It has three corners and three sides."
> Child: "It is equilateral."
> Teacher: "Yes, what does that mean?"
> Child: "It has equally long sides."
> Teacher: "How about the angles?"

One of the children took the triangle and checked whether the corners of the triangle would fit in one of the corners of the box.

> Child: "They are acute."
> Teacher, now showing the children a quadrilateral: "What can you tell me about this one? What is the difference between this one and the triangle?"
> Child: "It has four corners."
> Teacher: "How about the sides? Are there any parallel sides? You can check it out with these two rulers."

One of the children put rulers along two opposite sides of the quadrilateral.

> Child: "No!"
> Teacher: "How about the other two sides?"

[2] This time, the teacher had prepared a drawer with various quadrilaterals (quadrilateral, trapezium, parallelogram, rectangle, rhombus, and square) that had been taken from the second and fourth drawers in the Geometric Cabinet. For a description of the cabinet, see Ahlquist and Gynther (2019).

The child checked these sides and stated that they were not parallel either.

> Teacher: "What does parallel mean?"
> Child, illustrating this with the two rulers: "It is sort of like they don't go together."
> Teacher, pointing at the direction of the rulers: "Yes, they never meet each other even if they continue as far as we can see."
> Teacher, taking a card and reading the text on it: "It has four corners and four sides. Two sides are parallel. Which one of these [points at tray] could it be?"

The children looked at the tray, and one of them suggested the parallelogram.

> Teacher: "Does it have two parallel sides?"
> Child, checking with the rulers: "Yes!"
> Teacher: "How about these two sides then?"

The child checked them and stated that they were parallel as well.

> Teacher: "So, does it have two parallel sides?"

The children seemed to understand it was not the right one and then suggested the trapezoid, which they investigated with the rulers. The teacher then continued to read other cards for the children; finally, they had investigated all of the shapes and laid them on the carpet: quadrangle, trapezoid, parallelogram, rectangle, rhombus, and square.

> Teacher: "Now look—what is the difference between this one [indicating the quadrangle] and this one [indicating the trapezoid]?"
> Child, pointing at trapezoid: "This one has parallel sides, but this one [indicating the quadrilateral] doesn't."
> Teacher: "Exactly."

The teacher continued presenting the rectangle and square and then asked the children to describe the differences between the trapezoid and parallelogram, between the parallelogram and rectangle, and so on. Finally, the teacher asked the children to draw the quadrilaterals and write the names in their geometry books.

When looking back at the presentation described above, we noticed that the way the teacher gave her presentation differed from the way she had done it previously. This time, she began the presentation with a generative move by asking the children to describe the sides and angles of the triangle she held in her hand. In doing so, she enabled them to summarize what they had perceived in previous work with different kinds of triangles; consequently, in this case, the teacher was now aware whether the children understood what defined a right angle. This understanding was crucial when, for example, she later asked the children to describe what distinguishes a rhombus from a square.

Another example of a generative move was asking the children to explain the meaning of *parallel*, another critical aspect of identifying different quadrilaterals. It is reasonable that such generative moves, which were absent in the presentation described earlier, were used now because the teacher had reflected more on critical aspects.

In this presentation, the teacher also used contrast more, for example, by asking the children to investigate different shapes and describe their differences. Exposure to one specific quadrilateral lets children differentiate between the aspects that define the shape and those that do not. Other aspects, like color, remain invariant by the design of the material; therefore, according to variation theory, it is likely that the children will discern the aspect in focus.

The above illustration also creates conditions for the teacher to identify children's knowledge. In that way she will, during this presentation, direct their learning by using not only instructional moves but also confirming, reconstructing, and reorienting moves. In the presentation, the teacher confirmed the children's answers or actions either by statements or by moving on in the presentation. An example of a reconstructive move was the teacher asking the children to pay attention not only to the sides but also to the angles of an equilateral triangle. Finally, the teacher used a reorienting move when she had the children pay attention not only to the number of sides in a trapezoid but also to their relation to each other, another critical aspect of identifying and distinguishing different quadrilaterals.

However, awareness of critical aspects of the content is not the only necessary condition for successfully directing children's learning. Such directing must also be grounded in an awareness of the relationship between the children's knowledge and the intended learning object. According to the two presentations described, contrasting different shapes in a dialogue with the children created conditions for them to discern the different shapes and for the teacher to acknowledge the relationship between the intended learning object and what the children actually learned.

On the other hand, it is important to note that the task the children were supposed to work on independently (i.e., draw the shape and write its name) did not create those conditions. If the children had been asked not only to draw and write the names of the different quadrilaterals but also to describe what distinguishes each of them, it would likely have been easier for the teacher to identify whether each child needed to continue working to reach the intended learning object and if so, with what. Even if the children's individual work in this case did not create conditions for the teacher to identify critical aspects of the learning object each child had grasped, such conditions were created in other cases. For instance this was noticeable when some of the children in one of the sessions worked with a game using geometric solids, described next.

The Children's Individual Work With the Owl Game

During this observation, two boys were engaged in a game devised by Littler and Jirotková (2004), called the owl game, in which one boy asks yes–no questions of the other boy to find out what kind of solid the other boy had hidden under a cloth. The teacher had previously presented the solids, after which the boys performed a task in which they had to place the names of the solids under pictures of them.

When the game started, several shapes (i.e., a square pyramid, a triangular pyramid, a cube, a triangular prism, a rectangular prism, a sphere, an ovoid, an ellipsoid, a cone, a cylinder) were under a cloth. One of the boys, David, held a solid under the cloth without showing it to his classmate, Jonathan (both names are pseudonyms). Jonathan started asking questions to find out which solid David was holding in his hands. The researcher sat beside them, observing, and was able to intervene during their work. During the first section of the game, Jonathan had trouble imagining the hidden solid.

> Jonathan: "Is there a sharp edge?"
> David: "Yes."
> Jonathan: "Are there sides?"
> David: "Yes."
> Jonathan: "Is it a pyramid?"
> David: "No."
> Jonathan: "Is it a cube?"
> David: "No."
> Jonathan: "Is it a triangular prism?"
> David: "No."

David then showed Jonathan what he had been holding in his hand, a rectangular prism. Then Jonathan hid his hands under the cloth and held a solid. David asked if the solid had four sides, and Jonathan answered that it did not. David then asked if it had angles. When Jonathan affirmed it did, David asked if the angles were all equal. Jonathan reflected and said he did not think they were equal. David then started guessing.

> David: "Is it a square pyramid?"
> Jonathan: "No."
> David: "Is it a cube?"
> Jonathan: "No."
> David: "Is it, well, I don't know, maybe a cone?"
> Jonathan: "Yes."

The researcher, who had been watching the boys, asked them to remove the cloth.

> Researcher: "Can we have a look at the solids again?"

After the boys agreed, the researcher asked them to identify the solids with flat surfaces and the solids with curved ones. She told the boys to touch the different surfaces of the solids, identifying flat and curved ones. They touched the surfaces in the same way the researcher did, moving their fingers slowly along the different surfaces and explaining their experiences.

When they seemed to grasp the concept, the researcher asked them to place the solids into two groups according to whether they had flat or curved surfaces. Two solids remained, the cone and the cylinder. The researcher asked why they had not placed them in one of the two groups, and the boys replied that those solids were both flat and curved and should be placed in a third group. She then took the two solids from the group with flat surfaces (i.e., the square pyramid and the triangle pyramid), put them on their bases, picked up the square pyramid, and pointed at its base. She turned to Jonathan and asked him to identify the shape of the base. He answered that it was a square. She handed him the pyramid and asked how many sides the solid had. He immediately answered that it had four sides. The researcher asked him to look carefully and count them. He turned the solid around, recognized his mistake, and replied quickly without counting the sides: "Five," he said. She then turned to David and asked him how many triangles there were. He answered that there were four. "How do you know without counting them?" asked the researcher. "Because the base is a square," David replied. The researcher then asked him if he knew what the triangles were called. He shrugged his shoulders. Slowly the researcher marked the sides with her fingers and then moved the triangle in a walking motion (In Swedish, an isosceles triangle is *likbent*, meaning *with equally long legs*.) David smiled and said, "Oh, yes, it`s an isosceles triangle." Then the researcher picked out the triangular pyramid, pointed at its base, and handed it to Jonathan; she let the boys identify it in the same way they had with the square pyramid. This identification process went well and quickly.

Next, the researcher told the boys to ask as few questions as possible about one of the two pyramids, picking out the one with a triangular base. She put it in front of the other solids.

> Researcher: "Let's look at the surfaces. What is the first question you'd ask, David?"
> David: "Is the surface only flat?"
> Researcher: "Okay. What information would you get? Which of the solids can you skip?"
> David: "The ones with curved surfaces."

The researcher took away the solids with curved surfaces and placed the other solids with the pyramids.

> Researcher: "Now, what question could we ask here, Jonathan?"
> Jonathan: "Does it have a rectangular shape?"
> David: "No!"
> Researcher: "That's a good question—so, which of the solids can we take away?"

The boys took away the solids with rectangular shapes.

> Researcher: "Okay, now, here we have three solids. How can we make sure that we ask a question about the triangular pyramid?"
> David: "Are there only triangular shapes?"
> Researcher: "Okay. Jonathan, will you take away those that do not have only triangular shapes? You see, now there is only one solid that matches the question."

The two boys remained engaged and wanted to continue the game. The researcher asked them to identify the other solids before playing the game again. When they did, it appeared they were not sure of all the names of the shapes. The researcher asked them to look at their notebooks, where they had drawn the shapes and written the names, and encouraged them to make additional notes. Jonathan looked at his notes and asked what he should write. The researcher pointed at his drawing of a rectangle and asked him about the shape.

> Jonathan: "It's a rectangle."

The researcher asked him to define a rectangle. He answered correctly, and the researcher asked him to add the description to his notebook.

In the activity with solids, the two boys seemed able to visualize the solids because they grasped them and touched the surfaces with their hands. Grasping is crucial when teaching and learning geometry, according to Mwingirwa, Marguerite, and Khatete (2015), because many students lack spatial ability. The teacher therefore cannot expect that "his/her students are able to visualize figures, shapes and planes that may not be very obvious to the student" (p. 19).

When learning is seen as embodied, the material used in the owl game creates conditions that make it possible to connect the mind with the body, consequently, as Scoppola expressed in his preface to *Psychogeometry* (Montessori, 2011), ". . . making children 'perceive' deep relationships in order to 'prepare' the mind for the systematic study of the discipline . . . " (p. xvii). At the same time, it became clear in the activity that the material itself did not create such conditions unless the two boys had access to concepts necessary for this to happen. The task the boys had completed before the owl game (i.e., placing the names of the solids under a picture) did not seem to help much in regard to a systematic study of the solids and the relationships between them. A picture of a three-dimensional figure was problematic, as the boys seemed to lack a more profound, body-based experience. For example, when David asked questions and was supposed to conclude that Jonathan held a cone in his hand, he needed to be familiar with what constitutes flat and curved surfaces, concepts that he did not yet seem to understand as a feature that distinguishes the solids.

Analysis of the boys' work clearly shows they needed support. The goal of the owl game was to distinguish the solids, but simply being shown the material and then working with the pictures and labels did not help the boys succeed. To succeed in the owl game, they needed more body-based experience and guidance to distinguish certain critical aspects, which the researcher created by contrasting one solid with another. This reconstructing move showed a need for intervention, as it very soon became clear that the boys did not know how to distinguish the different solids. The researcher initially gave them some instructions, but then her epistemological moves were of a reconstructional nature, for instance, when she asked them to touch the different surfaces. Now they could distinguish the concept of *surface* and could place the solids in different categories according to their surfaces. The researcher confirmed their work by letting them move on as soon as she saw they knew what they were doing. There were also times when the researcher used reorienting moves, for instance, when Jonathan said that the square-based pyramid had four sides and he had the opportunity to reexamine the solid in question, thereby generating an explanation.

When we discussed this episode with the teacher, she realized that the children needed both more practice in identifying the different shapes and more time to understand certain concepts and characteristics through isolating, sorting, and classifying, activities they had not been able to do previously. At this point, it became obvious to the teacher that the Montessori material was essential, not only to establish concepts but also for the children to become aware of the shapes by bodily experiencing them. According to variation theory, it is essential in this work that the teacher organize the solids according to contrast, to make it possible for the children to discern the aspects in question. A first exercise, therefore, could be to let the children sort the solids using everyday objects, such as a ball for solids with surfaces without borders; a soup can, which has surfaces with and without borders; and a box, where all the surfaces have borders.

Discussion

In the previous sections, we showed how the intervention between the teacher and the researchers—for example, discussing what should be seen as critical in the content she was covering and the relation between such aspects and the observed teaching—seemed to have increased her awareness of the content-related aspects that she needed to address and clarify in her teaching and how she will do that. It also made her more aware of the relation between the children's actual knowledge and what they were supposed to learn. In variation theory, paying attention to what was conceptualized as the intended (i.e., planned), enacted (i.e., offered) and lived (i.e., discerned) learning object created conditions for the development of her teaching. For example, after we talked with the teacher about the owl game, she decided to present the blue solids differently next time. She would have the children pay attention to the names of the solids and their side surfaces, and she would let them hold the solids in their hands and focus on how different solids contrast with each other, giving them opportunities to make discoveries and arouse their interest. For instance, the children would contrast solids that had only curved surfaces with those that had only flat surfaces, as this was a critical aspect of the learning object for which they lacked the necessary concepts. In this way, according to variation theory and embodiment, the teacher would manage the intended learning object in a more powerful way.

As stated above, a prerequisite for such improvement is that the lived learning object become visible for the teacher. By reducing the number of children in the presentations and letting them play a more active role—letting each child, for example, describe the differences and similarities between geometrical shapes—it was possible for the teacher to identify how the children perceived the phenomenon. Such changes in the way the presentations were organized and conducted obviously not only gave the children an opportunity to adopt a more resonating and reflective attitude, but, by using different epistemological moves, it also allowed the teacher to direct the children's learning in a different way than before. However, these changes also presented challenges for the teacher in regard to the children who did not participate in the presentation. The teacher expressed this in one of the interviews:

> *I have to give presentations on more than one occasion about the same thing, and I feel that the presentations are carried out in a better way now. But that means I have less time to move around in the classroom and support the children in their work.*

In other words, when the number of children in the presentations was reduced, she needed to give the same presentation several times, which made it more challenging for her to support the children when they worked independently. This organizational change created a dilemma for her, which in itself indicated that the amount of support given in a more formal way (e.g., gathering all the children and following up on the lessons or giving written comments) was low. This low level of support was also confirmed in our observations. However, solving this dilemma with formal support presupposes that the tasks the children work on independently and then document in their workbooks are designed to make the relation between their knowledge and the intended learning object visible to the teacher. Such conditions were not created when the children worked independently with the different quadrilaterals. They only drew the shapes and wrote the names in their geometry books, which did not create conditions for the teacher to notice whether the children knew what defined each shape. If, instead, the children had written what they had discovered, their reasoning would have been visible, allowing the teacher to see how they perceived the learning object.

Whether the need for support is resolved formally or informally, the results of this study indicate that teaching needs to be designed so that teachers can better direct children's learning by using different epistemological moves, as shown in the owl game. However, as Lidar et al. (2006) mentioned, it is not enough to use the right epistemological move with a child. As in the owl game, there also has to be "a change in the students' practical epistemology, their learning of how and what to observe, [which] is a way of getting closer to the scientific concept" (Lidar et al., p. 13). According to Lithner (2015), rote learning and procedures will not solve learning difficulties in mathematics. For example, to analyze geometrical figures, children need concepts to describe them, such as angles, length, parallel sides, and so on. Knowing to describe a square as having four equally long sides and four right angles, for instance, makes it possible for children to precisely explain their knowledge of the figure.

> *Underlying most geometric thought is spatial reasoning, which is the ability to "see," inspect, and reflect on spatial objects, images, relationships, and transformations. Spatial reasoning includes generating images, inspecting images to answer questions about them, transforming and operating on images, and maintaining images in the service of other mental operations.* (Battista, 2007, p. 843)

Directions and practice in different learning situations will give children several learning opportunities. However, in the work to make the knowledge their own—in this case, to learn about geometric shapes and solids—it is essential that children be able to see critical aspects of a subject, to distinguish, for example, one solid from another and to grasp what characterizes each solid. As the results of this study show, this teacher's awareness of why a specific object of learning was to be treated in accordance with variation theory and embodiment seemed to help her successfully use epistemological moves in a more encouraging and constructive way.

Although this study is limited to one teacher and therefore cannot be generalized, it emphasizes the need for teachers to be aware of the motives underlying the ways their teaching is implemented if the conditions necessary for improvement are to be created. As the interventions between teacher and researchers in this study show, insights into variation theory and embodiment may help Montessori teachers deepen their awareness of such didactic motives.

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