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Mathematical Conversion of Section, Township, and Range Notation to Cartesian Coordinates

By Donald I. Good

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ABSTRACT

Section, township, and range notation provides sufficient information for accurate calculation of the coordinates of any location in an arbitrarily defined Cartesian coordinate system. First, such a set of calculations is derived; then, the results of these calculations are tested at 126 locations in Kansas. The mean absolute discrepancy for the x-coordinates (in Kansas) was found to be 0.447 mile and for the y-coordinates, 0.168 mile. Because of the complexity of the calculations involved and the number of figures that must be carried, the entire conversion from land grid notation to Cartesian coordinates was programmed in Fortran II for use on an IBM 1620 computer. A description of this program is also included.

INTRODUCTION

Currently there is increasing interest in mathematical applications in the field of geology. This interest is portrayed in many forms, but many of these involve some sort of mathematical representation of the geographic locations where geologic data have been observed. For purposes of mathematical treatment, it is usually convenient to represent these geographic locations as the coordinates of some two-dimensional coordinate system. One way of achieving this coordinate representation is simply to draw two arbitrary axes on a map of the geographic area being considered and physically measure the coordinates as established by these axes. This procedure of course often requires much tedious manual labor. There are, however, areas such as Kansas where land grid description quite commonly is used to describe geographic locations. Since this is a systematic method of description, it would seem that it should be possible to convert, by purely mathematical means, measurements of geographic location given in land grid description to mathematical coordinates. This conversion procedure, which is developed in the following pages, can then be programmed for a computer, thus saving the investigator a great deal of manual labor and time.

ACKNOWLEDGMENTS

Thanks are expressed to Dr. Floyd W. Preston of the Department of Chemical and Petroleum Engineering, The University of Kansas, for advice furnished in developing the conversion procedure and to Dr. Richard G. Hetherington of the Department of Mathematics and Director of the Computation Center of The University of Kansas, for reviewing this manuscript. The report was edited by Ada Swineford.

SELECTION OF THE COORDINATE SYSTEM

In much of the mathematical work that is currently of interest in geology, it is convenient to regard geographic locations where geologic data have been observed as though these locations were situated on an orthogonal, two-dimensional coordinate system. A question that naturally arises then is the following: If given a map of Kansas and the land grid description of some particular location, can Cartesian coordinates be calculated from this description so that if the appropriate coordinate axes were drawn on the map, then the calculated coordinates, when located with respect to these axes, would coincide with the position on the map of the described location? This question can be answered in the affirmative if a small error is permissible. Before this problem can be solved mathematically, however, two arbitrary selections must be made. First, a reference map must be selected; second, the axes of the coordinate system must be defined. The land grid description of any location in Kansas then may be converted to Cartesian coordinates with respect to the map and coordinate axes chosen.

The selection of a map is simply a matter of choice. A 1963 edition of a 1 : 500,000 USGS base map of Kansas (Lambert conformal conic projection; standard parallels 33° and 45°) was chosen primarily because this base is used for the 1964 edition of the Geologic Map of Kansas. Next, the location of the coordinate axes on this map is defined. The sixth principal meridian is defined to be the vertical axis, y-axis. The horizontal axis, x-axis, is determined by defining the origin of the coordinate system as the point of intersection of the y-axis and the boundary line between Ts. 35 S. and 36 S., this point being just south of the Kansas–Oklahoma line. The x-axis, therefore, is that line perpendicular to the y-axis which passes through the origin. The positive y direction is taken to be in the northerly direction; and the positive x direction, to be in the easterly direction (Fig. 1). Unity in this system is defined to be one mile. By using these coordinate axes all locations in west ranges have negative x-coordinates, all those in east ranges have positive x-coordinates, and all locations in Kansas have positive y-coordinates.

There is one objection to using the sixth principal meridian as a vertical axis. On the reference map the meridian is not drawn as a straight line, but appears to bend slightly to the east as one follows it southward from the Wichita area. The Bureau of Land Management of the Department of Interior was contacted concerning this discrepancy in the hope that precise measurements of the magnitude of this discrepancy were available. In the words of this agency (Personal communication, Clark L. Gumm, September 16, 1963),

> The Sixth Principal Meridian was established in 1855 along the longitudinal line 97° 22′ 08″, by private surveyors under contract with the General Land Office. This line was run with a solar compass in a general north–south direction. It is the opinion of this office that the meridian was not established to standards acceptable for present day accuracy requirements.

In order to obtain an estimate of this discrepancy, a straightedge and the map scale were used assuming that the maximum magnitude of the discrepancy was located at the south border of the state. By this method the maximum discrepancy is less than one-half mile; and hence, in succeeding calculations this discrepancy is disregarded.

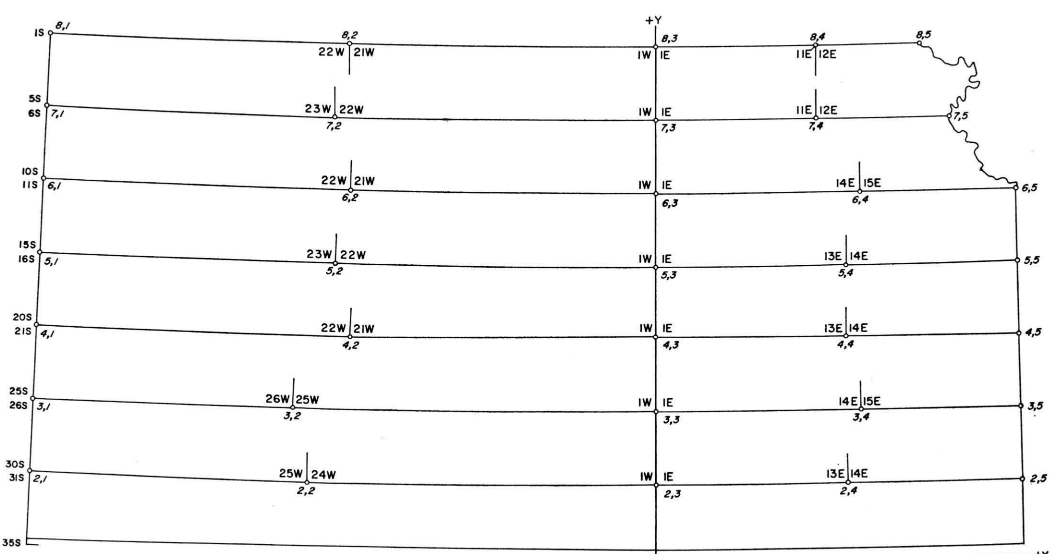

FIGURE 1.—Orientation of coordinate axes, location of correction lines in Kansas, and location of triangulated points. The triangulated points are indexed in the form p,q. The values of p and q are *not* the coordinates of the triangulated point but merely a systematic method of distinguishing one point from another. The value of p also serves to index the correction lines. The reason for omitting the value of $p = 1$ will become apparent later.

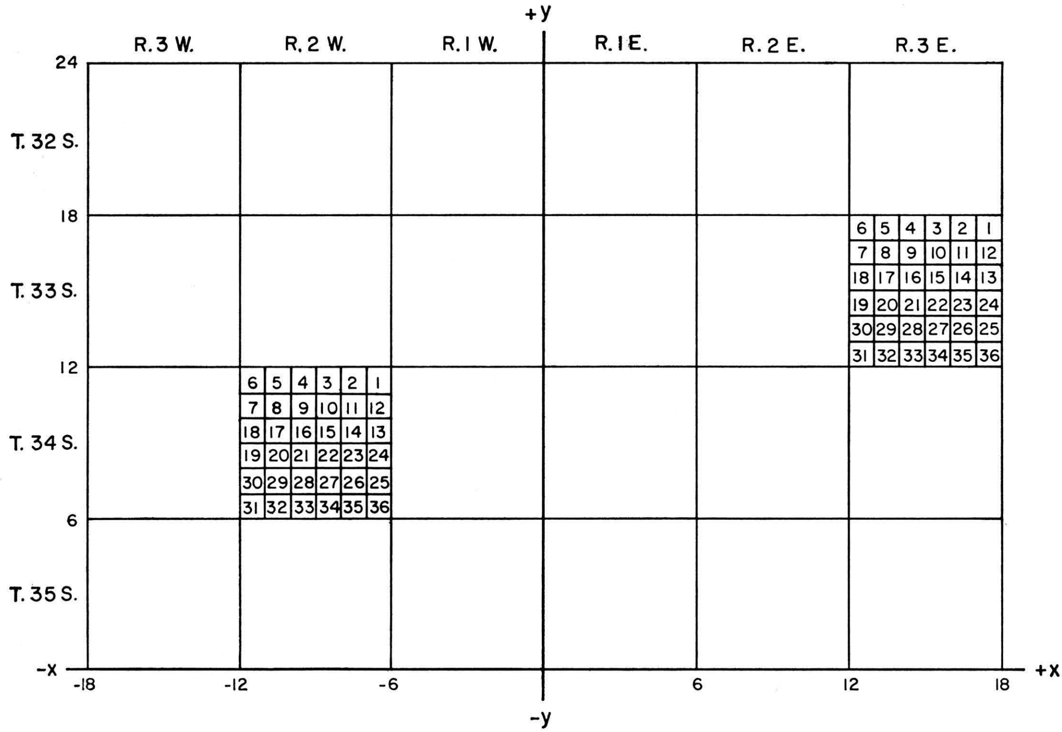

FIGURE 2.—Graphical representation of the basis on which the x^*y^*-coordinates are calculated.

It should also be noted that this particular selection of coordinate axes is by no means a necessity. These axes were selected merely because they are convenient for purposes of calculation. After coordinates have been calculated using this set of axes, the axes can be translated, rotated, or inverted to any desired position and the appropriate changes in the coordinates calculated by standard formulas.

DESCRIPTION OF THE CONVERSION

Now that a reference map and a set of coordinate axes have been selected the land grid notation can be converted mathematically to Cartesian coordinates. The essence of this conversion is first to calculate from the land grid description a pair of coordinates, (x^*, y^*), under the erroneous assumption that the sections, townships, and ranges are laid out on a flat, perfectly rectangular basis as in Fig. 2. These coordinates then are subjected to various corrections which will compensate for this false assumption.

The assumptions made to calculate the x^*y^*-coordinates are stated explicitly as follows: a) each section is exactly one mile square, b) the boundary lines between townships are straight lines parallel to the x-axis such that the boundary between Ts. 35 S. and 36 S. coincides with the x-axis, and c) the boundary lines between ranges are straight lines parallel to the y-axis such that the boundary between Rs. 1 E. and 1 W. coincides with the y-axis. Using these assumptions the point (x^*, y^*) is calculated.

Let us now assume that the land grid description of some particular location is given. This description will be allowed to be in the form

$$A_4B_4A_3B_3A_2B_2A_1B_1STRC$$

where A_iB_i is one pair of a set of at most four pairs of "quarter-section" notation (A_i is either N or S or absent; B_i is either E or W or absent; $i = 1, 2, 3, 4$),
S is the section number,
T is the township number,
R is the range number, and
C is the range direction. (C is either E or W.)

For example, the location NE¼, NE¼, NW¼, NW¼, sec. 18, T. 21 S., R. 3 E., would be written in the form

$$NENENWNW182103E.$$

Since all locations in Kansas are in south townships a township direction is not needed to specify a location uniquely; hence, it is not used. It should be noted here that by allowing only that form of quarter-section notation given above to enter into the calculations the description of a location is limited only slightly. If a location is described in some other way, this description may be transformed to four pairs of quarter-section notation with a maximum loss of accuracy of 0.03125 mile. When it is desired to describe sections which obviously are not one mile square, such as those in the westernmost tier in range 8 E. (Schoewe, 1948, p. 276), these sections should be divided into false quarters by bisecting the section with respect to its length and width. For further subdivisions of the false quarters the same procedure should be used. It is also assumed in the quarter-section notation that if the pair A_jB_j is absent, then any other pair A_kB_k is also absent if $k > j$. This is in keeping with recognized conventions used in land grid notation.

To determine y^*, the first step is to calculate a value L_T which is the y^*-coordinate of the southern boundary of the township T. This value is given by the equation

$$L_T = 6(35 - T).$$

Notice now that the y^*-coordinate of the center of the section S is always greater than L_T by a definite amount defined as S_T. It is obvious that S_T depends upon S. This relationship between S and S_T is shown in Table 1. (See also Fig. 2.)

TABLE 1.—Relationship between section number and the constant S_T.

Section Number						S_T
1,	2,	3,	4,	5,	6	5.5
7,	8,	9,	10,	11,	12	4.5
13,	14,	15,	16,	17,	18	3.5
19,	20,	21,	22,	23,	24	2.5
25,	26,	27,	28,	29,	30	1.5
31,	32,	33,	34,	35,	36	0.5

Now the amount that the quarter-section notation contributes to the y^*-coordinate of the location must be calculated. The y^*-coordinate of the desired location, which is assumed to be the center of the smallest "quarter" that is given in the description, differs from the y^*-coordinate of the center of the section S by a quantity arbitrarily called Q_T. By considering the quarter-section division of a section it can be seen that Q_T depends only on A_1, A_2, A_3, and A_4. The value of Q_T may be determined from Table 2. This determination is best explained by an example. Suppose that A_4 is absent (not given), A_3 is N, A_2 is N, and A_1 is S.

TABLE 2.—Determination of the constants Q_T and Q_{RW}.

	A_4	A_3	A_2	A_1	
N	1/32	1/16	1/8	1/4	E
S	−1/32	−1/16	−1/8	−1/4	W
Absent	0	0	0	0	Absent
	B_4	B_3	B_2	B_1	

To determine these values corresponding to A_1, A_2, A_3, and A_4 the top row and the left column of Table 2 are used as indices so that

$$Q_T = 0 + \frac{1}{16} + \frac{1}{8} - \frac{1}{4} = -\frac{1}{16},$$

which asserts that the y^*-coordinate of the described location is 1/16 mile below the center of section S. Thus, the complete calculation of y^* is given by

$$y^* = L_T + S_T + Q_T. \qquad \text{Eq. 1}$$

Using a similar procedure x^* is calculated. Here, however, different calculations are made for west ranges than are made for east ranges. Consider first the west ranges.

To begin, the x^*-coordinate of the western boundary of the range R is found. This value, L_{RW}, is given by

$$L_{RW} = -6R.$$

The center of the section S differs from L_{RW} by a certain quantity defined as S_{RW}. Here again S_{RW} depends on S. The relationship between S_{RW} and S is shown in Table 3. (See also Fig. 2.)

Now a third quantity, Q_{RW}, which is analogous to Q_T, is determined from Table 2. In this case only the values of B_4, B_3, B_2, and B_1 affect Q_{RW}, and the bottom row and the extreme right column of Table 2 are used as indices. For instance, if B_4 is absent, B_3 is E, B_2 is W, and B_1 is E, then

$$Q_{RW} = 0 + \frac{1}{16} - \frac{1}{8} + \frac{1}{4} = \frac{3}{16}.$$

TABLE 3.—Relationship between section number and the constant S_{RW}.

Section Number						S_{RW}
1,	12,	13,	24,	25,	36	5.5
2,	11,	14,	23,	26,	35	4.5
3,	10,	15,	22,	27,	34	3.5
4,	9,	16,	21,	28,	33	2.5
5,	8,	17,	20,	29,	32	1.5
6,	7,	18,	19,	30,	31	0.5

Thus, the x^*-coordinate for west ranges, x_W^*, is given by

$$x_W^* = L_{RW} + S_{RW} + Q_{RW} . \quad \text{Eq. 2}$$

Now consider the x^*-coordinate for locations in east ranges, x_E^*. This coordinate is found in the same way as x_W^* except that a different formula must be used to calculate the x^*-coordinate of the west boundary of the range R. This value, L_{RE}, is given by

$$L_{RE} = 6(R-1).$$

Now since

$$S_{RE} = S_{RW} \quad \text{and}$$
$$Q_{RE} = Q_{RW}, \quad \text{hence}$$
$$x_E^* = L_{RE} + S_{RW} + Q_{RW} . \quad \text{Eq. 3}$$

(The quantities S_{RE} and Q_{RE} bear the same relation to L_{RE} as S_{RW} and Q_{RW} bear to L_{RW}.)

This expression for x_E^*, however, does not take into account the fact that some sections in R. 8 E. are significantly more than one mile wide in the east–west direction. In the westernmost tier of sections in R. 8 E. the northern boundary of the northernmost section measures 5,395.5 feet in the east–west direction, and the southern boundary of the southernmost section of this tier measures 13,332 feet in the east–west direction (Schoewe, 1948, p. 279). Thus, the center of each section in this tier lies farther east than it would if the sections were exactly one mile wide as was assumed. If x_a^* is the x^*-coordinate of the center of a one mile square section and x_b^* is the x^*-coordinate of the actual center of this section, then the discrepancy, Δx^*, can be represented by

$$\Delta x^* = x_b^* - x_a^*.$$

If Δx_n^* is the discrepancy in miles at the north boundary of the state and Δx_s^* is the discrepancy in miles at the south boundary, then

$$\Delta x_n^* = 42.5109375 - 42.5000000$$
$$= 0.0109375 \text{ mile} \quad \text{Eq. 4}$$
$$\Delta x_s^* = 43.2625000 - 42.5000000$$
$$= 0.7625000 \text{ mile} \quad \text{Eq. 5}$$

In order to approximate this discrepancy, Δx^*, by a simple method it is assumed that Δx^* is a linear function of y^*, this function being expressed as

$$\Delta x^* = f(y^*) = my^* + b . \quad \text{Eq. 6}$$

Since the value of y^* at the north boundary of the state is 210 miles and y^* at the south boundary is 3 miles, Eq. 6 asserts that

$$0.0109375 = f(210) = 210m + b \quad \text{and} \quad \text{Eq. 7}$$
$$0.7625000 = f(3) = 3m + b . \quad \text{Eq. 8}$$

Solving Eqs. 7 and 8 simultaneously for m and b, the relationship between Δx^* and y^* as defined by Eq. 6 is

$$\Delta x^* = 0.77339221 - 0.0036307367y^*.$$

Now, Eq. 3 may be improved by using the relation

$$x_E^* = L_{RE} + S_{RW} + Q_{RW} + \Delta x^* \quad \text{Eq. 9}$$

if the condition that $\Delta x^* = 0$ for any location west of R. 8 E. is imposed. Note that Eq. 9 is still not exact but it is a closer approximation of x_E^* than is Eq. 3.

At this point the coordinates, (x^*, y^*), of a location can be calculated using Eqs. 1, 2, and 9 if it is assumed that the sections, townships, and ranges are laid out on a perfectly rectangular basis with a correction for the obvious violation of this assumption in R. 8 E. Now calculations must be made that will compensate for the false assumptions made so far.

In looking at the reference map, one will observe that in Kansas there are seven "correction lines." To be specific these are the following boundaries: the north boundary of the state, the boundary lines between Ts. 5 S. and 6 S., 10 S. and 11 S., 15 S. and 16 S., 20 S. and 21 S., 25 S. and 26 S., and 30 S. and 31 S. After some consideration, it becomes clear that these lines closely resemble segments of very large circles. If it is assumed that each correction line is an arc of some particular circle, this assumption, although not exactly true, furnishes a very good approximation and also an effective method for correcting the x^*y^*-coordinates.

In subsequent calculations it is necessary to know the equations of the circles that are to represent the correction lines. To determine these equations five points on each correction line were selected as shown in Fig. 1. However, before the equations of the circles could be found it was necessary first to determine as precisely as possible the true coordinates of each of the points shown in Fig. 1. These coordinates were determined using triangles. Two reference points, (x_1, y_1) and (x_2, y_2), were selected on the reference map. The point (x_1, y_1) was the origin of the coordinate system and point (x_2, y_2) was the point of intersection of the y-axis with the north boundary of Kansas. A third point (x_3, y_3) was then selected as shown in Fig. 3. This third point represents *any* of the points as shown in Fig. 1. The distances E, F, and G which are shown in Fig. 3 were then measured to the nearest 1/32 of an inch (about one-fourth mile). Then, using the law of co-

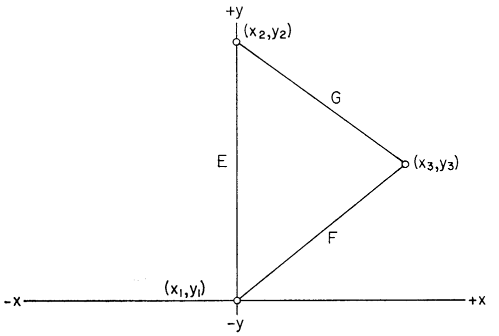

FIGURE 3.—Determination, by use of triangles, of the xy-coordinates of the points shown in Fig. 1.

sines, the Pythagorean theorem, and certain trigonometric definitions,

$$x_3 = \pm\sqrt{(2EF)^2 - (E^2 + F^2 - G^2)} \quad \text{and}$$
$$y_3 = \frac{E^2 + F^2 - G^2}{2E}.$$

The positive root for x_3 was used if the point (x_3, y_3) is to the right of the y-axis and the negative root was used if the point is to the left. In this way the xy-coordinates for each point were calculated. These coordinates are given in Table 4.

It is known that if three points, (x_i, y_i), (x_j, y_j), and (x_k, y_k) are given and that if these points are not colinear, then there is a unique circle that passes through these three points. The equation of this circle is of the form

$$(x - a)^2 + (y - c)^2 = r^2$$

where a is the x-coordinate of the center of the circle,
c is the y-coordinate of the center of the circle, and
r is the radius of the circle.

Thus, the parameters a, c, and r may be determined by solving simultaneously the system of equations

$$(x_i - a)^2 + (y_i - c)^2 = r^2$$
$$(x_j - a)^2 + (y_j - c)^2 = r^2$$
$$(x_k - a)^2 + (y_k - c)^2 = r^2.$$

This system of equations was solved three times for each correction line each time using a different set of three points. To illustrate this procedure, consider the points indexed 2,1, 2,2, 2,3, 2,4, and 2,5 in Fig. 1. These are the points that lie on the southernmost correction line whose index $p = 2$. The above system of equations was solved first letting (x_i, y_i) be the coordinates of the point indexed 2,1; (x_j, y_j), the coordinates of the point 2,3; and (x_k, y_k), the coordinates of the point 2,5. The system of equations was solved a second time letting (x_i, y_i) be the coordinates of the point indexed 2,1; (x_j, y_j), the coordinates of point 2,3; (x_k, y_k), the coordinates of point 2,4. The system of equations was solved a final time letting (x_i, y_i) be the coordinates of the point indexed 2,2; (x_j, y_j), the coordinates of point 2,3; (x_k, y_k), the coordinates of point 2,5. Proceeding in a similar manner, three values for a, c, and r were calculated for each of the other correction lines shown in Fig. 1. These values are given in Table 5. After calculating each of the parameters, a, c, and r three times on correction line p, these three values were averaged numerically to obtain the values a_p, c_p, and r_p. It then was assumed that the circle given by the equation

$$(x - a_p)^2 + (y - c_p)^2 = r_p^2$$

is a close approximation of the correction line p.

In subsequent calculations it is also necessary to know the equation of the circle representing the correction line ($p = 1$) which passes through the origin of the coordinate system. Since it appears that the correction line circles are approximately concentric, the value of a_1 was taken to be the numerical average of values of $a_2, a_3, \ldots, a_8$ and likewise the value of c_1 to be the numerical average of $c_2, c_3, \ldots, c_8$. Since the circles appear to be approximately concentric an averaging of the radii obviously would be invalid. But since it is known that this circle passes through the origin, r_1 can be determined from the equation

$$(0 - a_1)^2 + (0 - c_1)^2 = r_1^2.$$

The parameters of each of the circles that are to represent the correction lines are given in Table 6.

The remainder of the conversion now can be outlined with the aid of Fig. 4. In Fig. 4A the line P represents the first correction line that lies south of the point (x^*, y^*). Therefore, as can be seen from Fig. 4A, the point (x^*, y^*) can be located by starting at the intersection of P and the y-axis, moving a distance x^* along the line P, and then moving a distance h along the line perpendicular to P at the point $(x^*, y^* - h)$. In Fig. 4B the same procedure is used to locate the desired point (x, y) except that P is no longer considered a straight line but rather a segment of one of the circles whose parameters were determined previously. With this modification the point (x, y) can be located by starting

TABLE 4.—Descriptions and triangulated coordinates of test locations.

LOCATION		DESCRIPTION	X-CORRECT	Y-CORRECT	X-TRIANGULATED	Y-TRIANGULATED
2,1[1]	1[2]	NWNWNWNW0531S43W	-0.8438[3]	+0.03125[4]	-255.616215[5]	35.636796[6]
2,1	2	NWNENWNW0631S43W	0.	+0.03125	-255.616215	35.636796
2,1	3	SWSWSWSW3330S43W	-0.4063	-0.03125	-255.616215	35.636796
2,1	4	SWSESWSE3230S43W	0.	-0.03125	-255.616215	35.636796
2,2	5	SESESESE3630S25W	+0.03125	-0.03125	-143.815844	31.367373
2,2	6	SWSWSWSW3130S24W	-0.03125	-0.03125	-143.815844	31.367373
2,2	7	NWNENWNW0131S25W	0.	+0.03125	-143.815844	31.367373
2,2	8	NENENENE0631S24W	-0.88125	+0.03125	-143.815844	31.367373
2,3	9	SWSWSWSW3130S01E	-0.03125	-0.03125	0.000000	29.883921
2,3	10	SESESESE3630S01W	+0.03125	-0.03125	0.000000	29.883921
2,3	11	NENENENE0131S01W	+0.03125	+0.03125	0.000000	29.883921
2,3	12	NWNWNWNW0631S01E	-0.03125	+0.03125	0.000000	29.883921
2,4	13	SWSWSWSW3130S14E	-0.03125	-0.03125	79.336023	30.395220
2,4	14	SESESESE3630S13E	+0.03125	-0.03125	79.336023	30.395220
2,4	15	NWNWNWNW0631S14E	+0.09900	+0.03125	79.336023	30.395220
2,4	16	NWNENWNW0631S14E	0.	+0.03125	79.336023	30.395220
2,5	17	SESESESE3530S25E	+0.78125	-0.03125	150.813401	31.715382
2,5	18	SWSWSESE3630S25E	+0.03125	-0.03125	150.813401	31.715382
2,5	19	NWNWNENE0131S25E	+0.03125	+0.03125	150.813401	31.715382
3,1	20	SWSWSWSW3425S43W	-0.3688	-0.03125	-254.194645	65.644205
3,1	21	NWNWNWNW0426S43W	-0.4938	+0.03125	-254.194645	65.644205
3,1	22	NWNWNWNE0526S43W	0.	+0.03125	-254.194645	65.644205
3,1	23	SWSESWSE3325S43W	0.	-0.03125	-254.194645	65.644205
3,2	24	SWSWSWSW3125S25W	-0.03125	-0.03125	-149.495486	61.806914
3,2	25	SESESESE3625S26W	+0.03125	-0.03125	-149.495486	61.806914
3,2	26	NENENENE0126S26W	-0.61875	+0.03125	-149.495486	61.806914
3,2	27	NENWNENW0126S26W	0.	+0.03125	-149.495486	61.806914
3,3	28	SWSWSWSW3125S01E	-0.03125	-0.03125	0.000000	59.520869
3,3	29	SESESESE3625S01W	+0.03125	-0.03125	0.000000	59.520869
3,3	30	NENENENE0126S01W	+0.03125	+0.03125	0.000000	59.520869
3,3	31	NWNWNWNW0626S01E	-0.03125	+0.03125	0.000000	59.520869
3,4	32	SWSWSWSW3125S15E	-0.03125	-0.03125	84.981437	60.337751
3,4	33	SESESESE3625S14E	+0.03125	-0.03125	84.981437	60.337751
3,4	34	NWNWNWNW0626S15E	+0.1937	+0.03125	84.981437	60.337751
3,4	35	NWNWNENW0626S15E	0.	+0.03125	84.981437	60.337751
3,5	36	SESESESE3525S25E	+0.2213	-0.03125	150.076644	61.693195
3,5	37	SWSWSESW3625S25E	0.	-0.03125	150.076644	61.693195
3,5	38	NENENENE0126S25E	+0.03125	+0.03125	150.076644	61.693195
4,1	39	NWNWNWNW0321S43W	-0.4188	+0.03125	-252.750917	96.193675
4,1	40	NENWNWNE0421S43W	0.	+0.03125	-252.750917	96.193675
4,1	41	SESWSWSW3520S43W	0.	-0.03125	-252.750917	96.193675
4,2	42	SWSWSWSW3120S21W	-0.03125	-0.03125	-125.701774	90.857479
4,2	43	SESESESE3620S22W	+0.03125	-0.03125	-125.701774	90.857479
4,2	44	NENENENE0221S22W	+0.28750	+0.03125	-125.701774	90.857479
4,2	45	NWNWNENW0121S22W	0.	+0.03125	-125.701774	90.857479
4,3	46	SWSWSWSW3120S01E	-0.03125	-0.03125	0.000000	89.651765
4,3	47	SESESESE3620S01W	+0.03125	-0.03125	0.000000	89.651765
4,3	48	NENENENE0121S01W	+0.03125	+0.03125	0.000000	89.651765
4,3	49	NWNWNWNW0621S01E	-0.03125	+0.03125	0.000000	89.651765
4,4	50	SWSWSWSW3120S14E	-0.03125	-0.03125	79.050623	90.322271
4,4	51	SESESESE3620S13E	+0.03125	-0.03125	79.050623	90.322271
4,4	52	NENENENE0121S13E	+0.03125	+0.03125	79.050623	90.322271
4,4	53	NENENENW0621S14E	0.	+0.03125	79.050623	90.322271
4,5	54	SESESESE3420S25E	+0.60268	-0.03125	149.358319	91.871621
4,5	55	NENENENE0221S25E	+0.60268	+0.03125	149.358319	91.871621
4,5	56	SWSWSWSE3520S25E	0.	-0.03125	149.358319	91.871621
4,5	57	NENENENW0121S25E	0.	+0.03125	149.358319	91.871621
5,1	58	NWNWNWNW0116S43W	-0.8313	+0.03125	-251.347479	125.214790
5,1	59	NENENWNW0216S43W	0.	+0.03125	-251.347479	125.214790
5,1	60	SWSWSWSW3115S42W	+0.2813	+0.03125	-251.347479	125.214790

[1] Entries in this column correspond to locations shown in Fig. 1.

[2] Descriptions are numbered in sequence.

[3] That amount which should be added to the calculated x-coordinate to obtain the triangulated x-coordinate.

[4] That amount which should be added to the calculated y-coordinate to obtain the triangulated y-coordinate.

[5] Triangulated x-coordinate of locations shown in Fig. 1.

[6] Triangulated y-coordinate of locations shown in Fig. 1.

TABLE 4.—continued

5,1	61	SESESWSE3615S43W	0.	-0.03125	-251.347479	125.214790
5,2	62	SWSWSWSW3115S22W	-0.03125	-0.03125	-131.720016	120.924373
5,2	63	SESESESE3615S23W	+0.03125	-0.03125	-131.720016	120.924373
5,2	64	NWNWNWNW0616S22W	-0.9063	+0.03125	-131.720016	120.924373
5,2	65	NWNENWNW0116S23W	0.	+0.03125	-131.720016	120.924373
5,3	66	SWSWSWSW3115S01E	-0.03125	-0.03125	0.000000	119.535687
5,3	67	SESESESE3615S01W	+0.03125	-0.03125	0.000000	119.535687
5,3	68	NENENENE0116S01W	+0.03125	+0.03125	0.000000	119.535687
5,3	69	NWNWNWNW0616S01E	-0.03125	+0.03125	0.000000	119.535687
5,4	70	SWSWSWSW3115S14E	-0.03125	-0.03125	78.536628	120.386685
5,4	71	SESESESE3615S13E	+0.03125	-0.03125	78.536628	120.386685
5,4	72	NWNWNWNW0516S14E	-0.87500	+0.03125	78.536628	120.386685
5,4	73	NWNENWNW0616S14E	0.	+0.03125	78.536628	120.386685
5,5	74	SESESESE3415S25E	+0.13709	-0.03125	148.435616	122.012431
5,5	75	SWSWSWSW3515S25E	0.	-0.03125	148.435616	122.012431
5,5	76	NENENENE0316S25E	+0.54869	+0.03125	148.435616	122.012431
5,5	77	NWNWNWNE0216S25E	0.	+0.03125	148.435616	122.012431
6,1	78	NWNWNWNW0611S42W	-0.7188	+0.03125	-249.931901	155.568897
6,1	79	NWNWNENW0111S43W	0.	+0.03125	-249.931901	155.568897
6,1	80	SWSWSWSW3210S42W	-0.1938	+0.03125	-249.931901	155.568897
6,1	81	SESWSESE3110S42W	0.	-0.03125	-249.931901	155.568897
6,2	82	SWSWSWSW3110S21W	-0.03125	-0.03125	-125.585584	151.250049
6,2	83	SESESESE3610S22W	+0.03125	-0.03125	-125.585584	151.250049
6,2	84	NENENENE0211S22W	+0.03125	+0.03125	-125.585584	151.250049
6,2	85	NWNWNENW0111S22W	0.	+0.03125	-125.585584	151.250049
6,3	86	SWSWSWSW3110S01E	-0.03125	-0.03125	0.000000	149.666584
6,3	87	SESESESE3610S01W	+0.03125	-0.03125	0.000000	149.666584
6,3	88	NENENENE0111S01W	+0.03125	+0.03125	0.000000	149.666584
6,3	89	NWNWNWNW0611S01E	-0.03125	+0.03125	0.000000	149.666584
6,4	90	SWSWSWSW3110S15E	-0.03125	+0.03125	84.256698	150.394532
6,4	91	SESESESE3610S14E	+0.03125	-0.03125	84.256698	150.394532
6,4	92	NENENENE0111S14E	+0.22529	+0.03125	84.256698	150.394532
6,4	93	NWNENWNW0611S15E	0.	+0.03125	84.256698	150.394532
6,5	94	SESESESE3410S25E	+0.15473	-0.03125	148.012207	151.643547
6,5	95	NENENENE0311S25E	+0.59573	+0.03125	148.012207	151.643547
6,5	96	SESWSWSW3510S25E	0.	-0.03125	148.012207	151.643547
6,5	97	NWNENWNE0211S25E	0.	+0.03125	148.012207	151.643547
7,1	98	SWSWSWSW3305S42W	-0.03125	-0.03125	-248.571800	185.569745
7,1	99	NWNWNWNW0506S42W	-0.51563	+0.03125	-248.571800	185.569745
7,1	100	NWNWNWNE0606S42W	0.	+0.03125	-248.571800	185.569745
7,2	101	SWSWSWSW3105S22W	-0.03125	-0.03125	-131.883556	181.067344
7,2	102	SESESESE3605S23W	+0.03125	-0.03125	-131.883556	181.067344
7,2	103	NWNWNWNW0106S23W	-0.53125	+0.03125	-131.883556	181.067344
7,2	104	NWNWNWNE0206S23W	0.	+0.03125	-131.883556	181.067344
7,3	105	SWSWSWSW3105S01E	-0.03125	-0.03125	0.000000	179.550506
7,3	106	SESESESE3605S01W	+0.03125	-0.03125	0.000000	179.550506
7,3	107	NENENENE0106S01W	+0.03125	+0.03125	0.000000	179.550506
7,3	108	NWNWNWNW0606S01E	-0.03125	+0.03125	0.000000	179.550506
7,4	109	SWSWSWSW3105S12E	-0.03125	-0.03125	65.926752	179.956250
7,4	110	SESESESE3605S11E	+0.03125	-0.03125	65.926752	179.956250
7,4	111	NENENENE0106S11E	+0.12500	+0.03125	65.926752	179.956250
7,4	112	NENWNWNW0606S12E	0.	+0.03125	65.926752	179.956250
7,5	113	SESESESE3605S20E	+0.27821	-0.03125	120.018820	181.193601
7,5	114	NENENENE0106S20E	+0.71333	+0.03125	120.018820	181.193601
7,5	115	SWSWSESW3105S21E	0.	-0.03125	120.018820	181.193601
7,5	116	NENENWNE0606S21E	0.	+0.03125	120.018820	181.193601
8,1	117	NWNWNWNW0401S42W	-0.2063	+0.03125	-247.147418	215.240663
8,1	118	NENWNENE0501S42W	0.	+0.03125	-247.147418	215.240663
8,2	119	NENENENE0601S21W	-0.03125	+0.03125	-125.208270	210.587948
8,2	120	NWNWNWNW0101S22W	+0.03125	+0.03125	-125.208270	210.587948
8,3	121	NWNWNWNW0601S01E	-0.03125	+0.03125	0.000000	209.187453
8,3	122	NENENENE0101S01W	+0.03125	+0.03125	0.000000	209.187453
8,4	123	NENENENE0101S11E	+0.03125	+0.03125	65.692728	209.761151
8,4	124	NWNWNWNW0601S12E	-0.03125	+0.03125	65.692728	209.761151
8,5	125	NENENENE0601S19E	+0.77801	+0.03125	108.164173	210.707790
8,5	126	NWNWNENE0501S19E	0.	+0.03125	108.164173	210.707790

TABLE 5.—The three values for a, c, and r obtained for each correction line.

p	a	c	r
2	2.372982	5817.064411	5787.180975
	4.169524	5896.893753	5867.011313
	–4.247977	6590.041106	6560.158560
3	–5.897985	5093.866825	5034.349411
	–0.850126	5303.412044	5243.891244
	2.200341	5092.668364	5033.147976
4	–1.441321	4919.833760	4830.182209
	1.320569	5026.541223	4936.889634
	–8.904056	5714.483855	5624.839136
5	–14.211617	5055.500169	4935.984940
	–10.708354	5210.548660	5091.024234
	–11.618938	5265.121573	5145.599003
6	–2.644512	5332.287128	5182.621219
	2.108221	5533.540391	5383.874220
	3.637252	5419.064117	5269.398788
7	1.082863	5359.815370	5180.264977
	–6.571393	5043.723529	4864.177461
	–8.437909	5180.061953	5000.518566
8	–8.285209	4919.345932	4710.165765
	–10.716889	4820.062475	4610.887476
	–10.895625	4832.778572	4623.603956

TABLE 6.—The indices and parameters of the correction line circles.

p	a_p	c_p	r_p
1	–4.216198	5306.030700	5306.032200
2	0.764813	6101.333090	6071.450283
3	–1.515893	5163.315744	5103.796210
4	–3.008269	5220.286279	5130.636993
5	–12.179636	5177.056801	5057.536059
6	1.033653	5428.297212	5278.631409
7	–4.642146	5194.533617	5014.987001
8	–9.965908	4857.395660	4648.219066

at the intersection of P and the y-axis, moving a distance x^* along P to the point $(\bar{x}, \bar{y})$, and then moving a distance h along the line perpendicular to P at the point $(\bar{x}, \bar{y})$. It might be asked now, "Why was the first correction line south of (x^*, y^*) chosen rather than the first one north?" The first line south was chosen because the land was surveyed from south to north. (It should also be noted at this point that by always using the correction line immediately south of (x^*, y^*), the eighth correction line will never be used in the calculations.)

To determine which correction line lies immediately south of the described location, y^* is divided by 30 and the result expressed in terms of an integer quotient, q, and a remainder, h.

$$\frac{y^*}{30} = q + h\,.$$

The value 30 is used as a divisor because the correction lines are spaced at 30-mile intervals along the y-axis. The index, p, of the circle representing the correction line immediately south of (x^*, y^*) is found from the equation

$$p = q + 1$$

and the remainder h determined above is exactly the h shown in Fig. 4.

Now the coordinates of the point $(\bar{x}, \bar{y})$ can be calculated. The point $(\bar{x}, \bar{y})$ is the point on circle p that lies x^* units from the y-axis if x^* is measured along the circle. This distance is measured either east or west of the y-axis depending on whether x^* is positive or negative. Thus, $\bar{x}$ can be found by evaluating the expression

$$\int_0^{\bar{x}} ds = x^* \qquad \text{Eq. 10}$$

where ds denotes an infinitesimal unit of length along the arc of circle p. Now since

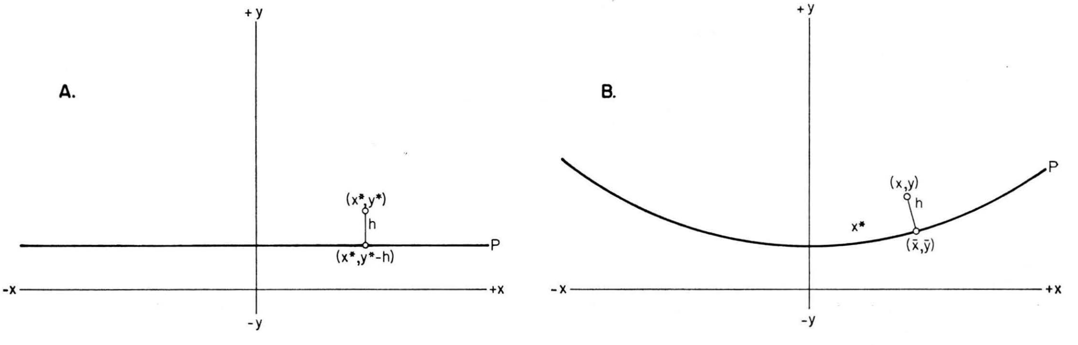

FIGURE 4.—The correction of the x^*y^*-coordinates.

$$f_p(x, y) = (x - a_p)^2 + (y - c_p)^2 - r_p^2 = 0 \qquad \text{Eq. 11}$$

is the equation of circle p, and

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

where dy/dx is found from Eq. 11, Eq. 10 becomes

$$\int_0^{\bar{x}} \sqrt{1 + \frac{(x - a_p)^2}{r_p^2 - (x - a_p)^2}}\, dx = x^*;$$

thus,

$$r_p \int_0^{\bar{x}} \frac{dx}{\sqrt{r_p^2 - (x - a_p)^2}} = x^*$$

so that

$$r_p \left[\sin^{-1}\left(\frac{\bar{x} - a_p}{r_p}\right) - \sin^{-1}\left(\frac{-a_p}{r_p}\right) \right] = x^*$$

and finally

$$\bar{x} = a_p + r_p \sin\left[\frac{x^*}{r_p} + \sin^{-1}\left(\frac{-a_p}{r_p}\right) \right] = x^*.$$

However it was found that the form

$$\bar{x} = a_p + \left[\frac{r_p^2}{\sqrt{r_p - a_p}} \right] \left[\sin\left(\frac{x^*}{r_p}\right) \right] - a_p \cos\left(\frac{x^*}{r_p}\right) \qquad \text{Eq. 12}$$

was more efficient for purposes of calculation. When the value of $\bar{x}$ is determined, $\bar{y}$ can be determined by substitution into Eq. 11. Thus,

$$\bar{y} = c_p - \sqrt{r_p^2 - (\bar{x} - a_p)^2}. \qquad \text{Eq. 13}$$

Now it is assumed that (x, y), the rectangular coordinates of the location, lie h miles due north of $(\bar{x}, \bar{y})$. To simulate this situation mathematically the line normal to circle p at the point $(\bar{x}, \bar{y})$ is determined, and then a distance h is measured northward from $(\bar{x}, \bar{y})$ along this normal line. One of the simplest ways of handling this situation is by use of vectors. If the equation of circle p is given by Eq. 11, then a vector normal to circle p at any point (x, y) on the circle is given by the gradient of f_p,

$$\nabla f_p = 2(x - a_p)\boldsymbol{i} + 2(y - c_p)\boldsymbol{j}.$$

However, in the case of the correction line circles, the gradient vector points in a southerly direction from the circle rather than in a northerly direction as is desired. Also the magnitude of ∇f_p depends on a_p and c_p; therefore, it is convenient to find the vector $\boldsymbol{n}_p$ which is of unit length and in the opposite direction from ∇f_p.

Under these conditions $\boldsymbol{n}_p$ at the point $(\bar{x}, \bar{y})$ is given by

$$\boldsymbol{n}_p = \frac{(a_p - \bar{x})}{r_p}\boldsymbol{i} + \frac{(c_p - \bar{y})}{r_p}\boldsymbol{j}.$$

If $\boldsymbol{H}$ is the vector with tip at (x, y) and tail at $(\bar{x}, \bar{y})$, then $\boldsymbol{H}$ is given by

$$\boldsymbol{H} = h\boldsymbol{n}_p \quad \text{or}$$

$$\boldsymbol{H} = \frac{h(a_p - \bar{x})}{r_p}\boldsymbol{i} + \frac{h(c_p - \bar{y})}{r_p}\boldsymbol{j}. \qquad \text{Eq. 14}$$

But also

$$\boldsymbol{H} = (x - \bar{x})\boldsymbol{i} + (y - \bar{y})\boldsymbol{j}. \qquad \text{Eq. 15}$$

Therefore, by equating the $\boldsymbol{i}$ and $\boldsymbol{j}$ components of $\boldsymbol{H}$ in Eqs. 14 and 15 and solving for x and y,

$$x = \bar{x} + \frac{h}{r_p}(a_p - \bar{x}) \quad \text{and} \qquad \text{Eq. 16}$$

$$y = \bar{y} + \frac{h}{r_p}(c_p - \bar{y}). \qquad \text{Eq. 17}$$

These values, x and y, are the xy-coordinates of the location in question with respect to the coordinate system that has been defined and with respect to the assumptions made.

DISCREPANCIES BETWEEN CALCULATED AND TRIANGULATED COORDINATES

A simple test was made of the accuracy of the method of calculation described above. Since the xy-coordinates of 35 locations were already known, the conversion procedure was tested at these locations. The data used in the testing are given in Table 4. The "x-correct" and "y-correct" values are necessary because of the fact that the triangulated coordinates are located at boundary intersections and hence it is impossible to describe the location precisely using only quarter-section, section, township, and range notation. It is for this reason that several different descriptions are given for the same geographic location.

Because the triangulated points lie on the correction lines some question might arise as to the validity of using these points as test points. However, at each geographical location, at least one of the land grid descriptions lies just south of the correction line on which the triangulated point is located. Since it is the first correction line south of the described location rather than the nearest correction line that is used in the calculations, these locations immediately south of the correction line present a maximal testing condition rather than a minimal one.

The results of testing the calculated coordinates against the triangulated coordinates are given in Table 7. The "*x*-" and "*y*-discrepancies" were determined by adding to the calculated coordinate the correction value given in Table 4 and then subtracting this sum from the triangulated coordinate. The "*x*" and "*y* per cent" columns in Table 7 are the relative discrepancies, the discrepancy being first multiplied by 100 and then divided by the corresponding triangulated coordinate. In the "*x* percent" col-

TABLE 7.—Discrepancies in *x*- and *y*-coordinates.

MEAN X DISCREPANCY = .361874
MEAN Y DISCREPANCY = -.051855
MEAN ABSOLUTE X DISCREPANCY = .446937
MEAN ABSOLUTE Y DISCREPANCY = .167877
STANDARD DEVIATION OF X DISCREPANCIES = .482137
STANDARD DEVIATION OF Y DISCREPANCIES = .216447

LOCATION		X-DISCREPANCY	Y-DISCREPANCY	X PER CENT	Y PER CENT
2,1	1	.673900	-.346505	-.263637	-.972323
2,1	2	.699168	-.388007	-.273522	-1.088781
2,1	3	.681016	.326788	-.266421	.916996
2,1	4	.649378	.310925	-.254044	.872483
2,2	5	.169687	-.241940	-.117989	-.771311
2,2	6	.169705	-.240450	-.118001	-.766560
2,2	7	.217354	-.484045	-.151133	-1.543148
2,2	8	-.703035	-.436592	.488843	-1.391866
2,3	9	-.000003	.001070	0.000000	.003580
2,3	10	-.000005	.001062	0.000000	.003553
2,3	11	.023636	-.116220	0.000000	-.388904
2,3	12	.023990	-.116269	0.000000	-.389068
2,4	13	.674207	.012278	.849811	.040394
2,4	14	.674201	.013080	.849804	.043033
2,4	15	1.012787	-.247400	1.276578	-.813943
2,4	16	.987508	-.249344	1.244715	-.820339
2,5	17	.414725	.007596	.274992	.023950
2,5	18	.352475	-.012378	.233715	-.039028
2,5	19	1.231539	-.524301	.816597	-1.653144
3,1	20	1.037216	-.167191	-.408040	-.254692
3,1	21	.924544	.360958	-.363714	.549870
3,1	22	.927831	.339906	-.365008	.517800
3,1	23	1.042951	-.185819	-.410296	-.283069
3,2	24	.482686	.128540	-.322876	.207970
3,2	25	.482659	.126722	-.322858	.205028
3,2	26	.394758	.060793	-.264060	.098359
3,2	27	.397730	.045316	-.266048	.073318
3,3	28	.000010	.001101	0.000000	.001849
3,3	29	.000009	.001120	0.000000	.001881
3,3	30	-.003930	-.361989	0.000000	-.608171
3,3	31	-.003620	-.361982	0.000000	-.608159
3,4	32	.430607	.091951	.506707	.152393
3,4	33	.430598	.093005	.506696	.154140
3,4	34	.617221	-.120812	.726300	-.200226
3,4	35	.562179	-.124251	.661531	-.205925
3,5	36	.354117	-.061077	.235957	-.099001
3,5	37	.263056	-.070334	.175281	-.114006
3,5	38	.275307	-.027402	.183444	-.044416
4,1	39	1.044868	.419186	-.413398	.435772
4,1	40	1.060462	.397577	-.419568	.413308
4,1	41	1.053904	.410949	-.416973	.427210
4,2	42	.285749	-.265159	-.227322	-.291840
4,2	43	.285731	-.266657	-.227308	-.293489
4,2	44	.292402	-.196286	-.232615	-.216037
4,2	45	.269330	-.188656	-.214261	-.207639
4,3	46	.000019	.001579	0.000000	.001761
4,3	47	.000018	.001616	0.000000	.001802
4,3	48	.008717	.132017	0.000000	.147255
4,3	49	.009085	.131998	0.000000	.147234
4,4	50	.608006	.025912	.769134	.028688
4,4	51	.607998	.026904	.769124	.029786
4,4	52	1.076471	.180516	1.361748	.199857
4,4	53	.610718	.172708	.772565	.191213

TABLE 7.—continued

4,5	54	.363230	-.012016	.243193	-.013079
4,5	55	.248253	.133653	.166213	.145478
4,5	56	.403659	-.028646	.270262	-.031180
4,5	57	.354087	.118932	.237072	.129454
5,1	58	.894310	-.486672	-.355806	-.388669
5,1	59	.869803	-.526074	-.346055	-.420137
5,1	60	.250111	-.051890	-.099508	-.041440
5,1	61	.843559	-.004210	-.335614	-.003362
5,2	62	.268797	-.014930	-.204066	-.012346
5,2	63	.268780	-.016411	-.204054	-.013571
5,2	64	.388303	-.336097	-.294794	-.277939
5,2	65	.351616	-.358034	-.266941	-.296080
5,3	66	.000076	.000205	0.000000	.000171
5,3	67	.000075	.000355	0.000000	.000296
5,3	68	.017389	-.114457	0.000000	-.095751
5,3	69	.017755	-.114494	0.000000	-.095782
5,4	70	.203969	.055387	.259711	.046007
5,4	71	.203959	.056505	.259699	.046936
5,4	72	.527230	.079952	.671317	.066412
5,4	73	.522008	.093844	.664668	.077952
5,5	74	.019237	-.054383	.012959	-.044571
5,5	75	.093859	-.056366	.063232	-.046196
5,5	76	.485966	.145019	.327391	.118855
5,5	77	.475685	.128498	.320465	.105315
6,1	78	1.246908	.398360	-.498899	.256066
6,1	79	1.272828	.362969	-.509269	.233317
6,1	80	1.133454	-.173538	-.453505	-.111550
6,1	81	1.126939	-.119989	-.450898	-.077129
6,2	82	.401416	.056508	-.319635	.037360
6,2	83	.401397	.055004	-.319620	.036366
6,2	84	.724763	.432991	-.577106	.286274
6,2	85	.445443	.440036	-.354692	.290932
6,3	86	-.000005	.000686	0.000000	.000458
6,3	87	-.000007	.000674	0.000000	.000450
6,3	88	.071986	.131339	0.000000	.087754
6,3	89	.072356	.131189	0.000000	.087654
6,4	90	.031968	.010148	.037941	.006747
6,4	91	.031960	.073633	.037931	.048959
6,4	92	.410154	-.039041	.486790	-.025959
6,4	93	.449088	-.042596	.532999	-.028322
6,5	94	-.320012	-.073511	-.216206	-.048476
6,5	95	.195010	-.404769	.131752	-.266921
6,5	96	-.290233	-.076997	-.196087	-.050774
6,5	97	.107655	-.426484	.072733	-.281241
7,1	98	1.329051	.023825	-.534674	.012838
7,1	99	1.386708	-.076081	-.557870	-.040998
7,1	100	1.367672	-.099830	-.550212	-.053796
7,2	101	.102084	-.095539	-.077404	-.052764
7,2	102	.102063	-.097126	-.077388	-.053640
7,2	103	.841357	-.289592	-.637954	-.159936
7,2	104	.807107	-.302235	-.611984	-.166918
7,3	105	.000030	.001713	0.000000	.000954
7,3	106	.000028	.001771	0.000000	.000986
7,3	107	-.006046	-.115403	0.000000	-.064273
7,3	108	-.005691	-.115391	0.000000	-.064266
7,4	109	-.190259	-.090024	-.288591	-.050025
7,4	110	-.190265	-.089142	-.288600	-.049535
7,4	111	.084006	-.108148	.127423	-.060096
7,4	112	.084725	-.109682	.128513	-.060949
7,5	113	-.334353	.095949	-.278583	.052953
7,5	114	-.097281	.192884	-.081054	.106451

TABLE 7.—concluded

7,5	115	-.368544	.088167	-.307071	.048659
7,5	116	-.129502	.176013	-.107901	.097140
8,1	117	1.464562	-.269444	-.592586	-.125182
8,1	118	1.444419	-.278561	-.584436	-.129418
8,2	119	-.876584	-.394988	.700100	-.187564
8,2	120	.986276	-.441589	-.787708	-.209693
8,3	121	.027928	-.361327	0.000000	-.172728
8,3	122	.027553	-.361270	0.000000	-.172701
8,4	123	.105966	-.279741	.161305	-.133361
8,4	124	.106346	-.280616	.161883	-.133778
8,5	125	-.905082	-.118218	-.836766	-.056105
8,5	126	-.934509	-.136580	-.863972	-.064819

umn several values are given as 0.000000. These values are quite untrue. They arbitrarily were entered as zero whenever the triangulated x-coordinate was zero and hence the value for "x percent" was indeterminate.

SOURCES OF ERROR AND IMPROVEMENT OF CALCULATIONS

There are several probable sources of error in the calculation procedure. Probably the part of the procedure most subject to error is the determination of the equations of the functions that represent the correction lines. This determination is subject first to any errors already existing in the reference map; second, to errors of physical measurement; third, to round-off errors encountered in calculating the circle parameters from the measured values; fourth, to the error of approximating the correction lines by circles; and fifth, to error inherent in the method of determining the circle parameters.

Another problem is that of surveying error when the ground was originally surveyed. It would seem to me that, in general, accurate prediction of this error would be impossible. Exceptions to this generalization would be such instances as the surveying error encountered in R. 8 E.

There is also the problem of round-off error in the conversion procedure itself. The computer program of the procedure as presented here uses 15 figures in all floating point calculations. Originally 20 figures were used and, when reduced to 15, changes in the third decimal place occurred in some answers. From this it is assumed that use of 15 figures will result in three reliable decimal places. This assumption, however, is not a valid substitute for a rigorous error analysis of the round-off error.

There are at least two possible ways in which the calculation procedure could be modified so as to obtain more accuracy if more accuracy is necessary. First, a more precise method for determining the functions to represent the correction lines could be used. Second, it might be possible to determine an approximate functional relation between the coordinates and discrepancies such as those shown in Table 7. One possible method of determining this functional relation is the fitting of first, second, and third degree trend surfaces to the discrepancies. To elaborate, the x-discrepancy would be determined as a function of the calculated coordinates x and y, for instance $f(x, y)$; and the y-discrepancy would be determined as another function of x and y, for instance $g(x, y)$. Then after the values of x and y are calculated according to the previously described procedure, the x-coordinate could be corrected by adding the value of the x-discrepancy as calculated from the function $f(x, y)$. Likewise the y-coordinate could be corrected by use of the function $g(x, y)$. This technique was not employed in the calculation procedure described previously because it was thought that 126 discrepancies measured at essentially 35 locations distributed over the entire state of Kansas would not provide sufficient data for the least squares method commonly used in trend surfacing. However, if it is necessary to obtain greater accuracy for the xy-coordinates, it is possible that this method when used with adequate data would increase the accuracy of the calculated coordinates.

SUMMARY

It has been shown that it is possible to convert by mathematical methods land grid descriptions of geographic locations in the state of Kansas to the coordinates of a Cartesian coordinate system if errors of the magnitude of those given in Table 7 are tolerable. This conversion is accomplished by using Eqs. 1, 2, 9, 12, 13, 16, and 17. The use of these equations, however, would not be efficient without first programming them for a computer. Such a program, written in Fortran II and developed on an IBM 1620 computer, is included in the appendix.

APPENDIX: FORTRAN II PROGRAM FOR CONVERTING SECTION, TOWNSHIP, AND RANGE NOTATION TO CARTESIAN COORDINATES.

General Description

As can be seen from the nature of the mathematical calculations necessary to convert land grid notation to Cartesian coordinates, this conversion would not be feasible without the aid of a computer. The accompanying Fortran II program which performs this conversion has several features which should be pointed out. The maximum numbers of descriptions that can be processed by the program at one time is 395. A location identification is carried with each location description. The number of sets of quarter-section notation used may be 0, 1, 2, 3, or 4. The input data are checked for invalid characters and if present these characters are listed on the console typewriter and the invalid description is not processed. (Invalid characters in the description are any characters that are invalid for the state of Kansas.) A set of coordinates corresponding to the x^*y^*-coordinates may be punched if desired. These coordinates could be used in small areas which include no correction lines. A third coordinate, z, may be included at each locality if desired. This value may be any quantity which is to be used for further mathematical treatment of the xy-coordinates. No operations are performed on the z-coordinates. Finally, after one set of locations is processed a message is typed, the program executes a PAUSE, and then branches back to read in a new set of data.

The complete program is composed of the mainline and three function subprograms, STK, QF, and SRK. These subprograms perform operations equivalent to those of Tables 1, 2, and 3, respectively. These subprograms do not use a table look-up system but rather perform mathematical operations which produce a result equivalent to those of the tables. These mathematical operations are described only to the extent that they were tested and found to give the correct results.

This program was developed on an IBM 1620 computer with a 60,000 bit memory. Hence if it is desired to use this program on a computer of different memory size it may be necessary to change some of the source program statements. Decks of the source program will be available from the State Geological Survey of Kansas for a price of $5.00 each for a limited time.

NOMENCLATURE

Fortran II Variable	*Algebraic Equivalent*	*Remarks*
A	$a_p,\ p = 1, 2, \ldots, 7$	Array of the x-coordinates of the centers of the correction line circles.
C	$c_p,\ p = 1, 2, \ldots, 7$	Array of the y-coordinates of the correction line circles.
I		Subscript for locations.
ICHECK		Invalid character indicator. If ICHECK $= 0$, there are no invalid characters. If ICHECK > 0, there are invalid characters.
IM	q	
IP	p	
IQEW	B_4, B_3, B_2, B_1	IQEW(1) $= B_4, \ldots,$ IQEW(4) $= B_1$
IQNS	A_4, A_3, A_2, A_1	IQNS(1) $= A_4, \ldots,$ IQNS(4) $= A_1$
IRD	C	Range direction.
IRN	R	Range number in integer form.
ISN	S	Section number in integer form.
ITN	T	Township number in integer form.
K		Subscript index for quarter-section notation IQNS(K) or IQEW(K).
LOCA		Array of the first five characters of any valid notation that may be chosen to identify a location.
LOCB		Array of the second five characters of any valid notation that may be chosen to identify a location.
N		Number of locations for which coordinates are to be calculated.
NQ		The number of pairs of quarter-section notation used.
NS		Indicator for punching of x^*y^*-coordinates. NS $=$ 0, coordinates not punched; NS $= 1$, coordinates punched.
NZ		Indicator for punching of z-coordinates. NZ $= 0$, coordinates not punched; NZ $= 1$, coordinates punched.
QF		Fortran function for determining the values of Q_T or Q_{RW}.
R	$r_p,\ p = 1, 2, \ldots, 7$	Array of the radii of the correction line circles.
RL	L_R	
RN	R	Range number in decimal form.
RS	$\dfrac{r_p^2}{\sqrt{r_p^2 - a_p^2}} \sin \dfrac{x^*}{r_p}$	
RSQ	r_p^2	
SIM		IM in decimal form.
SN	S	Section number in decimal form.
SQ	$\sqrt{r_p^2 - a_p^2}$	
SRK		Fortran function for determining the value of S_R.
STK		Fortran function for determining the value of S_T.
T	h	
TL	L_T	
TN	T	Township number in decimal form.
X		Array of x-coordinates for all locations.
XA	Δx^*	
XB	$\bar{x}$	
XBASQ	$(\bar{x} - a_p)^2$	
XS	x^*	
Y		Array of y-coordinates for all locations.
YB	$\bar{y}$	
YS	y^*	
Z		Array of z-coordinates for all locations.

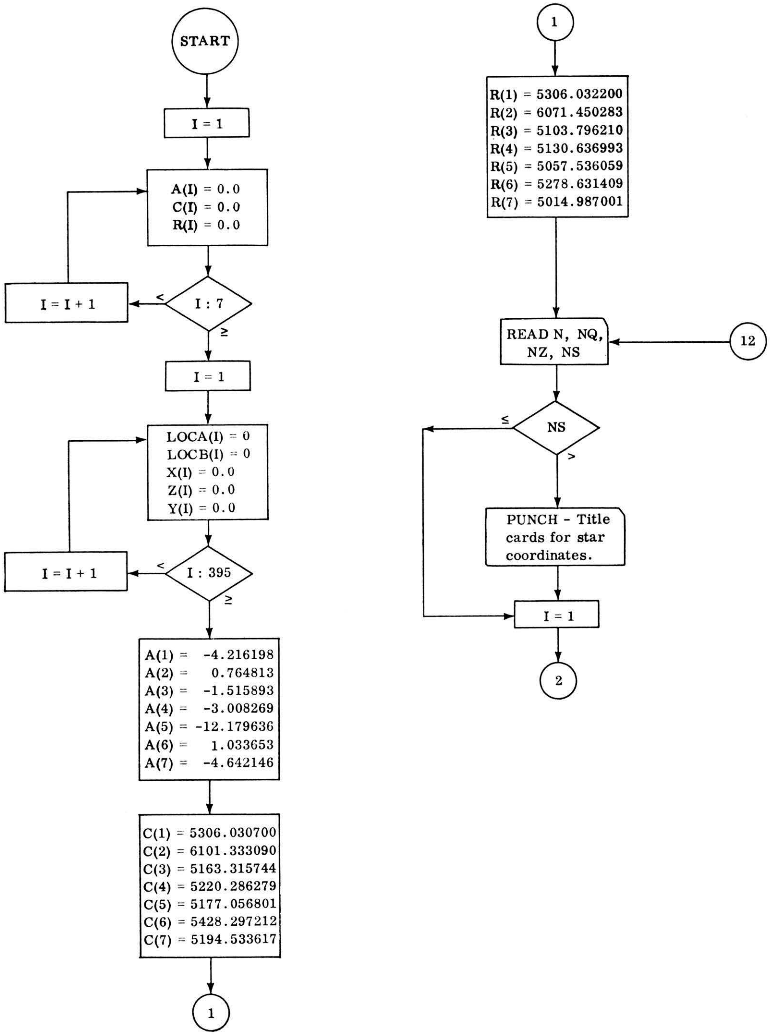

FIGURE 5.—Flow chart of mainline program.

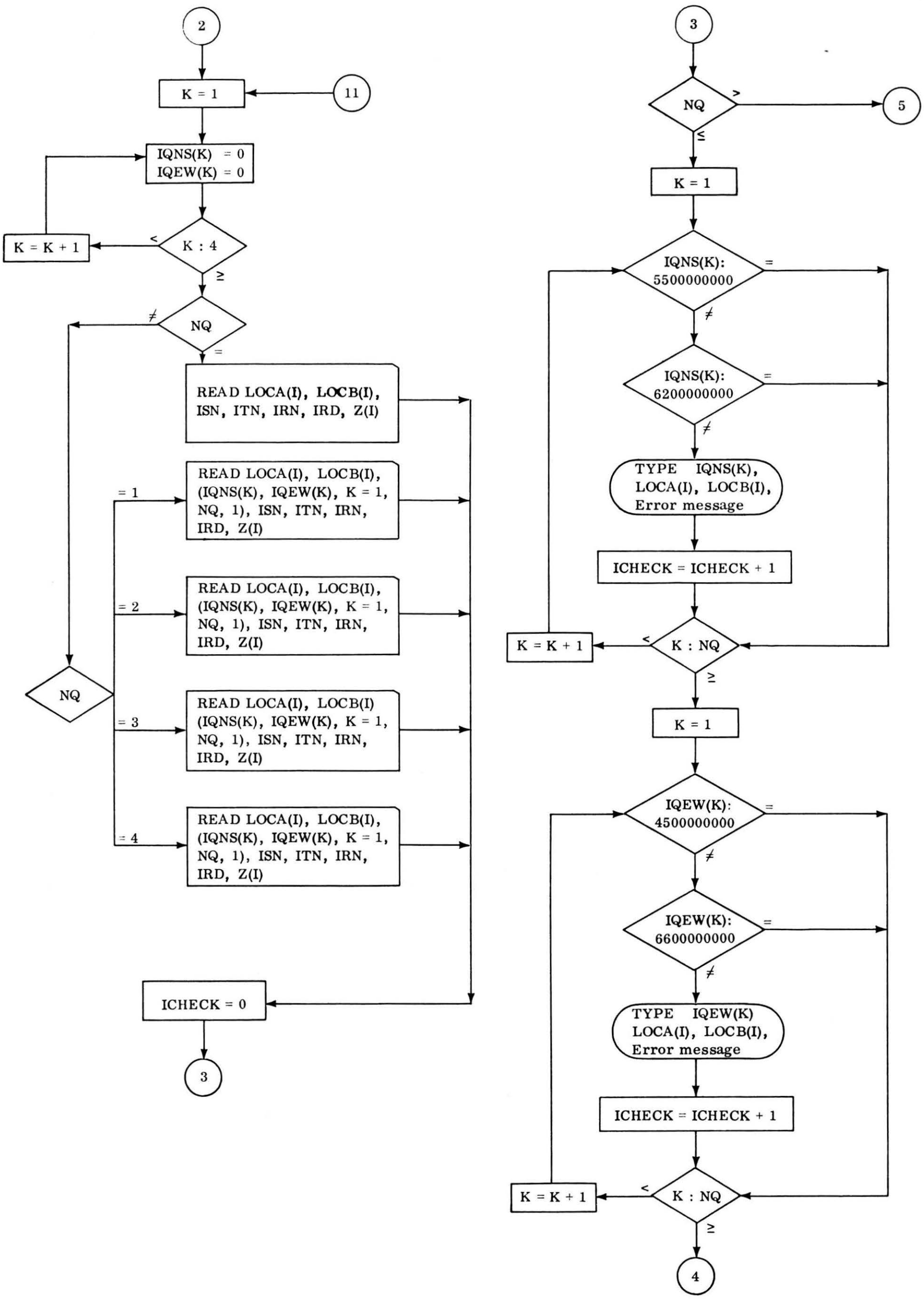

FIGURE 5.—continued

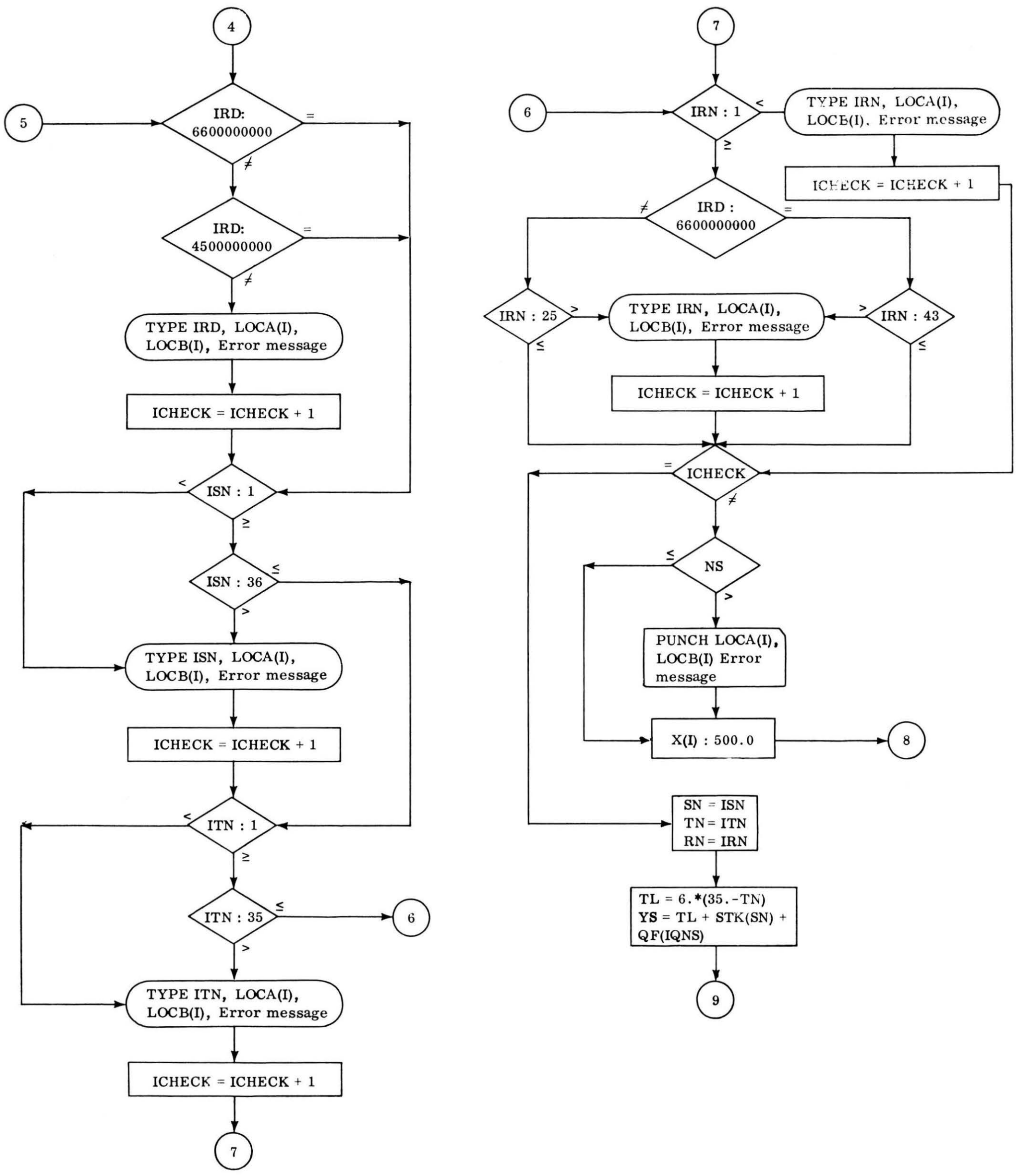

FIGURE 5.—continued

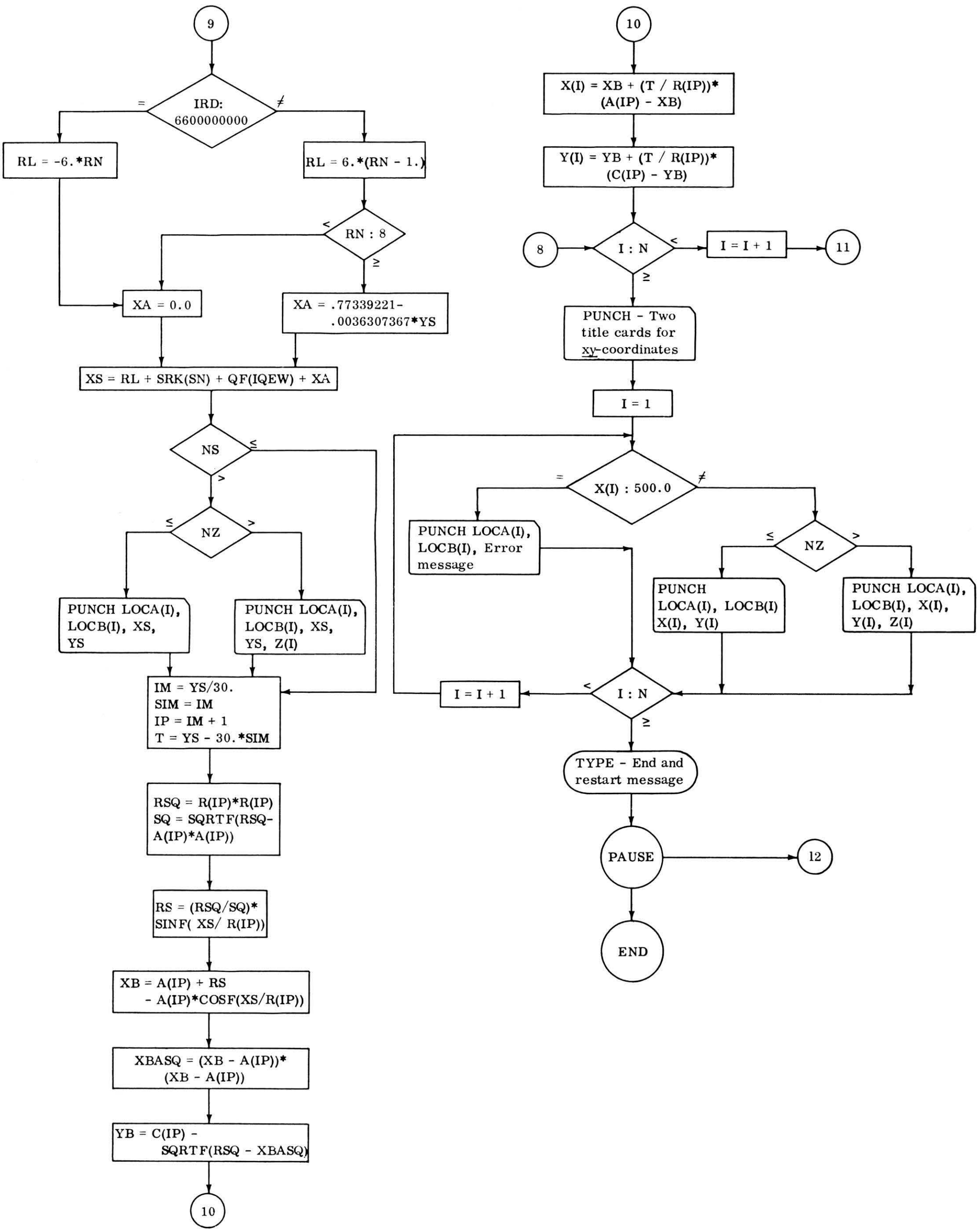

FIGURE 5.—concluded

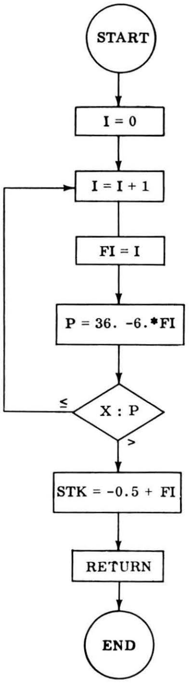

FIGURE 6.—Flow chart of function STK.

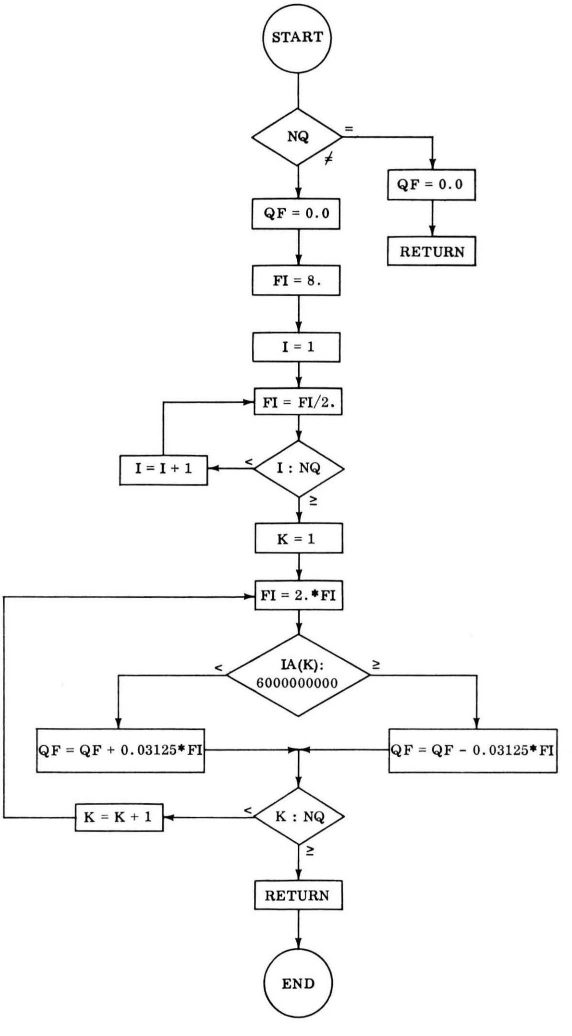

FIGURE 7.—Flow chart of function QF.

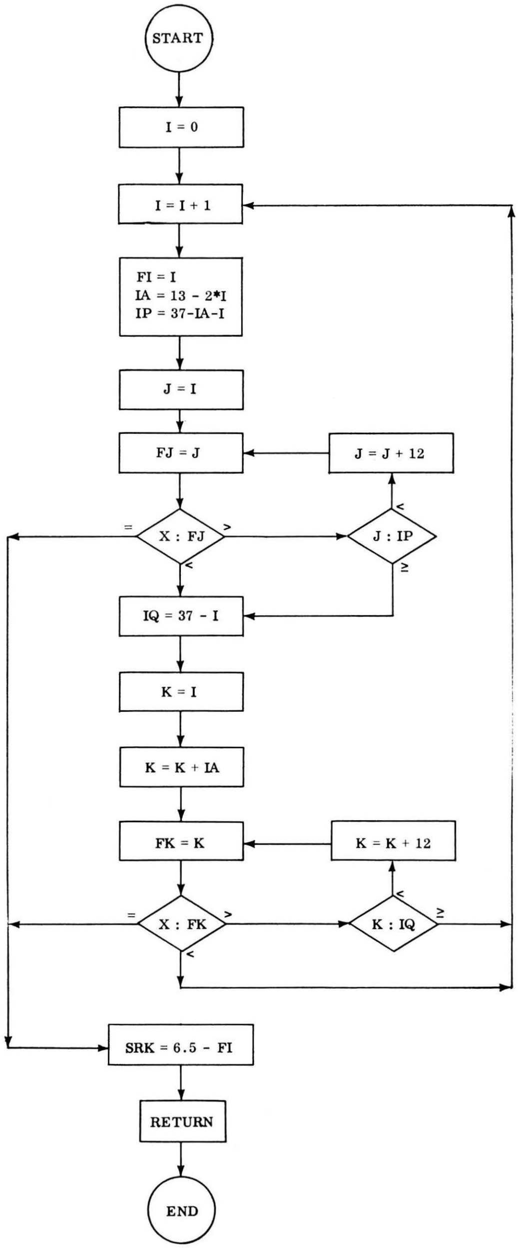

FIGURE 8.—Flow chart of function SRK.

THE FORTRAN II SOURCE PROGRAMS (Mainline and subprograms). See also Figs. 5 to 8.

```
*1510
C     THIS PROGRAM LOCATES ON A PLANAR RECTANGULAR COORDINATE SYSTEM
C     ANY LOCATION IN THE STATE OF KANSAS THAT IS GIVEN IN QUARTER-
C     SECTION, SECTION, TOWNSHIP, AND RANGE NOTATION.  IN THIS SYSTEM
C     THE SIXTH PRINCIPAL MERIDIAN IS DEFINED AS THE Y-AXIS.  THE
C     ORIGIN OF THE SYSTEM IS DEFINED AS THE INTERSECTION OF THE
C     BOUNDARY LINE BETWEEN TOWNSHIP 35S AND TOWNSHIP 36S WITH THE
C     Y-AXIS.  UNITY IN THE SYSTEM IS ONE MILE.
C
C     PROGRAMMER - DONALD I. GOOD
C     FOR - STATE GEOLOGICAL SURVEY OF KANSAS
C     DATE - 1963
C
      DIMENSION A(7), C(7), R(7), LOCA(395), LOCB(395), X(395), Y(395),
     1 IQNS(4), IQEW(4), Z(395)
      COMMON NQ
C     ------------------------------------------
C     ZERO A, C, R, LOCA, LOCB, X, Y, AND Z ARRAYS
C     ------------------------------------------
   15 DO 30 I = 1,7,1
   20 A(I) = 0.0
   25 C(I) = 0.0
   30 R(I) = 0.0
C
   35 DO 50 I = 1,395,1
   40 LOCA(I) = 0
   41 LOCB(I) = 0
   45 X(I) = 0.0
      Z(I) = 0.0
   50 Y(I) = 0.0
C     -------------------------
C     GENERATE CIRCLE PARAMETERS
C     -------------------------
      A(1) = -4.216198
      A(2) = 0.764813
      A(3) = -1.515893
      A(4) = -3.008269
      A(5) = -12.179636
      A(6) = 1.033653
      A(7) = -4.642146
C
      C(1) = 5306.030700
      C(2) = 6101.333090
      C(3) = 5163.315744
      C(4) = 5220.286279
      C(5) = 5177.056801
      C(6) = 5428.297212
      C(7) = 5194.533617
C
      R(1) = 5306.032200
      R(2) = 6071.450283
      R(3) = 5103.796210
      R(4) = 5130.636993
      R(5) = 5057.536059
      R(6) = 5278.631409
      R(7) = 5014.987001
C     ------------------------------------------------------------
C     READ IN NUMBER OF LOCATIONS, NUMBER OF SETS OF QUARTER-SECTION
C     NOTATION USED, AND INDICATORS FOR PUNCHING OF Z-COORDINATES
C     AND STAR COORDINATES
C     ------------------------------------------------------------
```

```
   70 READ 75, N, NQ, NZ, NS
   75 FORMAT (4I5 /)
C
      IF (NS) 91, 91, 85
   85 PUNCH 86
   86 FORMAT (1H1 20X, 16HSTAR COORDINATES)
      PUNCH 87
   87 FORMAT (1H0 10H  LOCATION 15H   X-COORDINATE 15H   Y-COORDINATE
     1 15H   Z-COORDINATE)
C
   91 DO 515 I = 1,N,1
C
   92 DO 94 K = 1,4,1
   93 IQNS(K) = 0
   94 IQEW(K) = 0
C     ---------------------------------------------------------
C     READ IN LOCATION IDENTIFICATION AND LOCATION DESCRIPTION
C     ---------------------------------------------------------
      IF (NQ) 95, 137, 95
   95 GO TO (100, 110, 120, 130), NQ
C
  100 READ 105, LOCA(I), LOCB(I), (IQNS(K), IQEW(K), K = 1,NQ,1), ISN,
     1 ITN, IRN, IRD, Z(I)
  105 FORMAT (2A5, 8X, 2A1, 2I2, 1X, I2, A1, F15.6)
  106 GO TO 139
C
  110 READ 115, LOCA(I), LOCB(I), (IQNS(K), IQEW(K), K = 1,NQ,1), ISN,
     1 ITN, IRN, IRD, Z(I)
  115 FORMAT (2A5, 6X, 4A1, 2I2, 1X, I2, A1, F15.6)
  116 GO TO 139
C
  120 READ 125, LOCA(I), LOCB(I), (IQNS(K), IQEW(K), K = 1,NQ,1), ISN,
     1 ITN, IRN, IRD, Z(I)
  125 FORMAT (2A5, 4X, 6A1, 2I2, 1X, I2, A1, F15.6)
  126 GO TO 139
C
  130 READ 135, LOCA(I), LOCB(I), (IQNS(K), IQEW(K), K = 1,NQ,1), ISN,
     1 ITN, IRN, IRD, Z(I)
  135 FORMAT (2A5, 2X, 8A1, 2I2, 1X, I2, A1, F15.6)
  136 GO TO 139
C
  137 READ 138, LOCA(I), LOCB(I), ISN, ITN, IRN, IRD, Z(I)
  138 FORMAT (2A5, 10X, 2I2, 1X, I2, A1, F15.6)
C     ----------------------------------------
C     CHECK FOR INVALID CHARACTERS AND NUMBERS
C     ----------------------------------------
  139 ICHECK = 0
  140 IF (NQ) 210, 210, 141
C
  141 DO 175 K = 1,NQ,1
  145 IF (IQNS(K) - 5500000000) 150, 175, 150
  150 IF (IQNS(K) - 6200000000) 160, 175, 160
  160 TYPE 165, IQNS(K), LOCA(I), LOCB(I)
  165 FORMAT (11HINVALID QS, 1X, A1, 13H, IN LOCATION 1X, 2A5)
  170 ICHECK = ICHECK + 1
  175 CONTINUE
C
  185 DO 205 K = 1,NQ,1
  190 IF (IQEW(K) - 4500000000) 195, 205, 195
  195 IF (IQEW(K) - 6600000000) 200, 205, 200
  200 TYPE 165, IQEW(K), LOCA(I), LOCB(I)
```

```
  201 ICHECK = ICHECK + 1
  205 CONTINUE
C
  210 IF (IRD - 6600000000) 215, 235, 215
  215 IF (IRD - 4500000000) 220, 235, 220
  220 TYPE 225, IRD, LOCA(I), LOCB(I)
  225 FORMAT (11HINVALID RD, 1X, A1, 13H, IN LOCATION 1X, 2A5)
  230 ICHECK = ICHECK + 1
C
  235 IF (ISN - 1) 245, 240, 240
  240 IF (ISN - 36) 260, 260, 245
  245 TYPE 250, ISN, LOCA(I), LOCB(I)
  250 FORMAT (11HINVALID SN, I3, 13H, IN LOCATION 1X, 2A5)
  255 ICHECK = ICHECK + 1
C
  260 IF (ITN - 1) 270, 265, 265
  265 IF (ITN - 35) 285, 285, 270
  270 TYPE 275, ITN, LOCA(I), LOCB(I)
  275 FORMAT (11HINVALID TN, I3, 13H, IN LOCATION 1X, 2A5)
  280 ICHECK = ICHECK + 1
C
  285 IF (IRN - 1) 305, 290, 290
  290 IF (IRD - 6600000000) 300, 295, 300
  295 IF (IRN - 43) 320, 320, 305
  300 IF (IRN - 25) 320, 320, 305
  305 TYPE 310, IRN, LOCA(I), LOCB(I)
  310 FORMAT (11HINVALID RN, I3, 13H, IN LOCATION 1X, 2A5)
  315 ICHECK = ICHECK + 1
C
  320 IF (ICHECK) 321, 329, 321
  321 IF (NS) 324, 324, 322
  322 PUNCH 323, LOCA(I), LOCB(I)
  323 FORMAT (1X, 2A5, 12X, 18HINVALID CHARACTERS)
  324 X(I) = 500.0
  325 GO TO 515
C     --------------------------
C     CALCULATION OF COORDINATES
C     --------------------------
  329 SN = ISN
  330 TN = ITN
  335 RN = IRN
C     ----------------
C     STAR COORDINATES
C     ----------------
  340 TL = 6. * (35. - TN)
  345 YS = TL + STK(SN) + QF(IQNS)
C
  350 IF (IRD - 6600000000) 365, 355, 365
  355 RL = -6. * RN
  360 GO TO 385
C
  365 RL = 6. * (RN - 1.)
C
  370 IF (RN - 8.) 385, 375, 375
  375 XA = .77339221 - .0036307367 * YS
  380 GO TO 390
C
  385 XA = 0.0
C
  390 XS = RL + SRK(SN) + QF(IQEW) + XA
C
```

```
      IF (NS) 435, 435, 400
  400 IF (NZ) 405, 405, 406
  405 PUNCH 545, LOCA(I), LOCB(I), XS, YS
      GO TO 435
  406 PUNCH 545, LOCA(I), LOCB(I), XS, YS, Z(I)
C
  435 IM  = YS / 30.
  440 SIM = IM
  445 IP = IM + 1
  450 T = YS - 30. * SIM
C     ---------------
C     BAR COORDINATES
C     ---------------
  455 RSQ = R(IP) * R(IP)
  460 SQ = SQRTF(RSQ - A(IP) * A(IP))
  465 RS = (RSQ / SQ) * SINF(XS / R(IP))
C
  470 XB = A(IP) + RS - A(IP) * COSF(XS / R(IP))
C
  475 XBASQ = (XB - A(IP)) * (XB - A(IP))
C
  480 YB = C(IP) - SQRTF(RSQ - XBASQ)
C     -----------------
C     FINAL COORDINATES
C     -----------------
  505 X(I) = XB + (T / R(IP)) * (A(IP) - XB)
  510 Y(I) = YB + (T / R(IP)) * (C(IP) - YB)
C
  515 CONTINUE
C     -----------------------
C     PUNCH FINAL COORDINATES
C     -----------------------
      PUNCH 525
  525 FORMAT (1H1 18X, 20HXYZ-COORDINATE TABLE)
      PUNCH 87
C
  530 DO 565 I = 1,N,1
C
  535 IF (X(I) - 500.0) 540, 555, 540
C
  540 IF (NZ) 541, 541, 551
  541 PUNCH 545, LOCA(I), LOCB(I), X(I), Y(I)
  545 FORMAT (1X, 2A5, 3F15.6)
  550 GO TO 565
  551 PUNCH 545, LOCA(I), LOCB(I), X(I), Y(I), Z(I)
      GO TO 565
C
  555 PUNCH 560, LOCA(I), LOCB(I)
  560 FORMAT (1X, 2A5, 8X, 18HINVALID CHARACTERS)
C
  565 CONTINUE
C
  570 TYPE 575
  575 FORMAT (/ 41HTHE COORDINATES FOR THIS SET OF DATA ARE 11HCALCULATE
     1D. / 42HTO PROCESS NEW SET OF POINTS, PRESS START.)
C
      PAUSE
      GO TO 70
      END
```

```
*1510
      FUNCTION STK(X)
C
    5 I = 0
   10 I = I +1
   15 FI = I
   20 P = 36. - 6. * FI
   25 IF (X - P) 10, 10, 30
C
   30 STK = -0.5 + FI
C
   40 RETURN
   45 END
```

```
*1510
      FUNCTION QF(IA)
C
      DIMENSION IA(4)
      COMMON NQ
C
   11 IF(NQ) 15, 12, 15
   12 QF = 0.0
      RETURN
C
   15 QF = 0.0
C
   20 FI = 8.
   25 DO 30 I = 1,NQ,1
   30 FI = FI / 2.
C
   60 DO 90 K = 1,NQ,1
C
   65 FI = 2. * FI
   70 IF (IA(K) - 6000000000) 75, 85, 85
C
   75 QF = QF + 0.03125 * FI
   80 GO TO 90
C
   85 QF = QF - 0.03125 * FI
C
   90 CONTINUE
C
      RETURN
      END
```

```
*1510
      FUNCTION SRK(X)
C
    5 I = 0
   10 I = I + 1
   15 FI = I
   20 IA = 13 - 2 * I
   25 IP = 37 - IA - I
C
   30 J = I
   35 FJ = J
   40 IF (X - FJ) 60, 100, 45
   45 IF (J - IP) 50, 60, 60
   50 J = J + 12
   55 GO TO 35
C
   60 IQ = 37 - I
   65 K = I
   70 K = K + IA
   75 FK = K
   80 IF (X - FK) 10, 100, 85
   85 IF (K - IQ) 90, 10, 10
   90 K = K + 12
   95 GO TO 75
C
  100 SRK = 6.5 - FI
C
  105 RETURN
  110 END
```

Form of Input

Card 1. This card contains the quantities N, NQ, NZ, and NS. Each of these four quantities must be in integer form. The quantity N may range from 1–395 and must be right-justified in columns 1–5; NQ may range from 0–4 and must be in column 10; NZ may be either 0 or 1 and must be in column 15. The quantity NS may be either 0 or 1 and must be in column 20. If it is desired that any of the quantities NQ, NZ, or NS be zero the appropriate columns may be left blank. It should be noted here that NQ, NZ, and NS remain fixed for the entire sequence of N locations. This is important to note when preparing the remainder of the cards beginning with card 3.

Card 2. This card is not read by the computer. It is included so that if it is so desired, the columns of input data may be labeled when listed on a tabulator. This card, however, must be present.

Card 3. This is the first card of a set of N cards each containing the necessary information for one location. The remainder of the N cards are to be punched in the same form as card 3.

Columns 1–10 contain the location identification. This identification may be any sequence of numbers, letters, or any characters acceptable to the computer.

Columns 11 and 12 are left blank.

Columns 13–20 contain the quarter-section notation (right-justified).

Columns 21–22 contain the section number in integer form (right-justified).

Columns 23–24 contain the township number in integer form (right-justified).

Column 25 is not read by the computer but is included so that a township direction may be recorded if desired.

Columns 26–27 contain the range number.

Column 28 contains the range direction.

Columns 29–43 contain the z-coordinate. The z-coordinate may be any number of 15 or fewer digits so long as the decimal point is punched on the card. If no decimal point is punched it will be assumed to be between columns 37 and 38. Columns 29–43 will always be read. For this reason these columns should be left blank if no z-coordinates are used.

Columns 44–80 are not read by the computer and hence may contain any additional desired information.

Form of Output

Both card and typewriter output are used in a given set of data. The card output is punched so as to tabulate with carriage control. (In the carriage control system, column 1 is used as an indicator for the tabulator. A blank indicates single space; a 0, double space; and 1, new page.) The output is best explained by examining a sample problem.

Sample Problems

Two sample problems are presented. In the first, ten land grid descriptions are to be converted to xy-coordinates. In this problem let us assume that 4 sets of quarter-section notation are to be used, that it is not desired to punch the x^*y^*-coordinates, and that it is not desired to punch the z-coordinates. The input for this problem should be in the following form (Table 8). (Notice that every character in the description of location ZILCH is invalid.)

First the card output for this problem and then the typewriter output for this problem are given below (Table 9).

TABLE 8.—Input for sample problem 1.

10 4

IDENT	DESCRIPTION	
A3 102	NWNESWSW1701S19E	2307.2
A3 100	NWNESWSE0814S06E	1.00
A3 101	SESESENW3420S17W	.9968
ABD6001	SWSWSWSW0130S01W	
ZILCH	ABCDEFGH3756S78I	
ABC + 10	NENWNWNE2306S33W	
A4 103	SWSWNENW1428S12E	70004
B4 204	NENENENE1814S27W	
C101	SWSESESW2118S03W	
T65	NENESENW0335S43W	

TABLE 9.—Output for sample problem 1.

Card Output

XYZ-COORDINATE TABLE

LOCATION	X-COORDINATE	Y-COORDINATE	Z-COORDINATE
A3 102	108.549908	208.049917	
A3 100	31.567326	129.929081	
A3 101	-98.515874	91.069649	
ABD6001	-.967313	34.914304	
ZILCH	INVALID CHARACTERS		
ABC + 10	-192.369112	180.196979	
A4 103	70.704599	46.067954	
B4 204	-160.717326	131.675517	
C101	-15.559323	103.695929	
T65	-254.168887	11.614176	

Typewriter Output

```
INVALID QS, A, IN LOCATION   ZILCH
INVALID QS, C, IN LOCATION   ZILCH
INVALID QS, E, IN LOCATION   ZILCH
INVALID QS, G, IN LOCATION   ZILCH
INVALID QS, B, IN LOCATION   ZILCH
INVALID QS, D, IN LOCATION   ZILCH
INVALID QS, F, IN LOCATION   ZILCH
INVALID QS, H, IN LOCATION   ZILCH
INVALID RD, I, IN LOCATION   ZILCH
INVALID SN, 37, IN LOCATION   ZILCH
INVALID TN, 56, IN LOCATION   ZILCH
INVALID RN, 78, IN LOCATION   ZILCH

THE COORDINATES FOR THIS SET OF DATA ARE CALCULATED.
TO PROCESS NEW SET OF POINTS, PRESS START.
```

In the typewriter output for invalid characters "QS" stands for "quarter-section" and is used for any part of the quarter-section notation; "RD" stands for "range direction"; "SN," for "section number"; "TN," for "township number"; and "RN," for "range number."

In the second sample problem let us assume that eight land grid descriptions are to be converted to *xy*-coordinates, that no quarter-section notation is to be used, that it is desired to punch the x^*y^*-coordinates, and that it is desired to punch the *z*-coordinates. The input for this problem should be in the form of Table 10.

First, the card output and then the typewriter output for this problem are given below (Table 11). If the card output is listed with carriage control, a new page is registered before the heading "XYZ-COORDINATE TABLE" is printed.

TABLE 10.—Input for sample problem 2.

8	1	1	
NAME		DESCRIPTION	Z-COORDINATE
AB - 1		1821S03E	10.00
BB - 10		2003S17W	15.00
AX - 2		0714S02E	6.0
BX - 7		3626S36W	20.0
TQ - 400		1010S10E	
NM - 407A		1228S22E	7673
PT3 - 4		2717S17W	42.3
DXX		1020S30W	

TABLE 11.—Output for sample problem 2.

Card Output

STAR COORDINATES

LOCATION	X-COORDINATE	Y-COORDINATE	Z-COORDINATE
AB - 1	12.500000	87.500000	10.000000
BB - 10	-100.500000	194.500000	15.000000
AX - 2	6.500000	130.500000	6.000000
BX - 7	-210.500000	54.500000	20.000000
TQ - 400	57.712443	154.500000	0.000000
NM - 407A	132.104562	46.500000	76.730000
PT3 - 4	-98.500000	109.500000	42.300000
DXX	-176.500000	94.500000	0.000000

XYZ-COORDINATE TABLE

LOCATION	X-COORDINATE	Y-COORDINATE	Z-COORDINATE
AB - 1	12.424463	87.038675	10.000000
BB - 10	-100.217108	194.960067	15.000000
AX - 2	6.461226	130.055166	6.000000
BX - 7	-209.605032	58.043236	20.000000
TQ - 400	57.663039	154.469832	0.000000
NM - 407A	131.737415	47.799484	76.730000
PT3 - 4	-98.131606	110.034532	42.300000
DXX	-176.314861	97.079734	0.000000

Typewriter Output

THE COORDINATES FOR THIS SET OF DATA ARE CALCULATED.
TO PROCESS NEW SET OF POINTS, PRESS START.

REFERENCE

SCHOEWE, W. H., 1948, The geography of Kansas; Part I, Political geography: Kansas Acad. Sci. Trans., v. 51, no. 3, p. 253–288.