DANIEL F. MERRIAM, Editor

PREDICTION OF THE PERFORMANCE OF A SOLUTION GAS DRIVE RESERVOIR BY MUSKAT'S EQUATION

By

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Northern Natural Gas Company

COMPUTER CONTRIBUTION 8

State Geological Survey

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Editor's Remarks

This publication, Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonio Baca is the first issue of the second year of the COMPUTER CONTRIBUTION series. The COMPUTER CONTRIBUTIONS, an outgrowth of a series which appeared in the Special Distribution Publications, apparently are useful as more than 10,000 copies have already been distributed.

Each year since 1963, when it was decided to publish computer programs and results of their use, it has been necessary to make changes in format and style of the publications. Each change, hopefully, has improved the presentation. This year, because of increased load, we have again enlarged the Board of Editors. We welcome J.C. Davis, J.E. Klovan, and R.A. Reyment to the Board. Three of the members, J.W. Harbaugh, W.C. Pearn, and F.W. Preston, have served since the beginning. The willingness of these people to serve brings a breadth of scope and background of experience to the Board that is unequaled anywhere.

COMPUTER CONTRIBUTION 8 should be of special interest to petroleum engineers. Prediction of reservoir performance is very important in extending the life of a producing field. Several modifications of the program are suggested by the author to enhance its usefulness; no doubt other modifications may be desirable or necessary to fit a particular need. According to the author "...The program described here will save many man-hours in predicting the performance of solution gas drive reservoirs as well as insure consistency of results." Two obviously desirable features of computer work are time saving and consistency.

> The advent of the electronic computer has created a new approach to engineering problems. The costly 'build and try' method is now often replaced by 'construct mathematical model and simulate'.
>
> IBM, 1961, General Information Manual an Introduction to Engineering Analysis for Computers, p. 4.

This program also, then, fulfills the notion of a new approach to engineering problems.

The Kansas Geological Survey is the only geological organization known to be actively distributing computer program decks as well as data decks. The programs are sold for a limited time at a nominal cost. Versions of the programs have been executed on Burroughs B5500, CDC 3400, Elliott 803C, GE 625, and IBM 1620, 7040, 7090, and 7094 computer systems. For a limited time, the Survey will make available the card deck of the program in ALGOL described in COMPUTER CONTRIBUTION 8 for $10.00. An up-to-date list of available decks can be obtained by writing, Editor, COMPUTER CONTRIBUTIONS, at the Survey offices in Lawrence.

Comments and suggestions concerning the COMPUTER CONTRIBUTION series are welcome and should be addressed to the Editor. An up-to-date list of publications is available on request.

PREDICTION OF THE PERFORMANCE OF A SOLUTION GAS DRIVE RESERVOIR BY MUSKAT'S EQUATION

By

APOLONIO BACA

ABSTRACT

This report describes a digital computer program using a method developed by M.M. Muskat in 1945 to predict the performance of a solution gas drive reservoir. The program is written in ALGOL for the Burroughs B5500 computer.

Many questions regarding recoverable reserves, proper field development, optimum production rates, and possible pressure maintenance can be predicted in the early stages of reservoir development. Information on pressure-volume-temperature (PVT) data, as well as some reservoir rock properties, are necessary to obtain data on oil saturation, producing gas-oil ratio, and cumulative oil and gas production as a function of reservoir pressure. These data then may be used to plot the reservoir performance curves.

INTRODUCTION

Regardless of driving force, it is necessary that future performance of a reservoir be known as early as possible, so that the operator may determine the proper field development, well spacing, and production, to insure maximum recovery from the field. It also will be of interest to know at which point in the history of the reservoir pressure maintenance may be required.

Coleman, Wilde, and Moore (1930) were the first to attempt predicting the performance of an oil reservoir. The significant contribution of this work is the presentation of a mathematical relation between the reservoir pressure, the amount of oil and gas produced, the amount of oil and gas content of the reservoir, and the properties of the reservoir fluids.

Schilthuis (1936) modified the early work and introduced his "active oil" principle. Schilthuis also introduced a form of the material balance equation which is in use today.

Buckley and Leverett (1942) discussed the various fluid displacement mechanisms in sands, and the advantages of water over gas as a displacing agent. Reasons cited for the inefficiency of solution gas drive are:

(1) Oil saturation decreases and the gas saturation increases simultaneously and more or less uniformly throughout the reservoir.

(2) Displacing fluid is able to compete for production on an equal basis with the oil.

(3) Because the gas is disseminated throughout the oil sand, it cannot be excluded by mechanical means from the oil-producing wells. Thus, while the gas is being produced, the reservoir energy is being dissipated until such time as no energy is left to expel the remaining oil.

Old (1943) illustrated the simultaneous use of the Schilthuis material balance equation, and the Hurst fluid flow equation to determine the magnitude of reserves and a water-drive parameter from pressure and production history. His work for predicting future performance involved a trial and error method using assumed pressures in the fluid flow equation until the material balance equation was satisfied. His study of the Schuler field in Arkansas shows how this method may be applied in the early producing life of a field provided accurate pressure and production data are available as well as bottom-hole sample analysis and values of porosity, net pay, and connate water.

Tarner (1944) discussed the use of the material balance equation and the instantaneous gas-oil ratio simultaneously to predict reservoir performance. Tarner combined all the variables into two independent equations and through a trial and error procedure determined the amount of gas and oil produced down to a particular reservoir pressure. It is necessary to assume several values of oil produced to some predetermined pressure and calculate the gas produced and plot the produced gas versus the produced oil. Assumed values of produced gas versus calculated values of produced oil also are plotted. The intersection of these two curves gives the value of oil and gas produced down to that pressure. Tarner does caution, however, against the utilization of this calculation method as a mechanical operation and emphasized that the results obtained necessarily depend on the judgment of the engineer and his familiarity with the particular problem.

Muskat (1945) expressed the material balance equation in differential form. He combined the material balance equation with the gas-oil ratio equation to produce the differential equation for oil saturation in terms of the pressure dependent variables in the material balance equation. This equation (derivation given in Appendix F) takes the form:

$$\frac{dS_o}{dp} = \frac{\frac{S_o}{B_g B_o}\frac{dR_s}{dp} + \frac{S_o}{B_o}\frac{k_g}{k_o}\frac{\mu o}{\mu g}\frac{dB_o}{dp} + (1-Sw-So)\frac{1}{B_g}\frac{dB_g}{dp}}{1 + \frac{k_g}{k_o}\frac{\mu o}{\mu g}} \qquad (1)$$

By numerically integrating this differential equation, the relation between residual oil saturation S_o and the reservoir pressure, p, is obtained directly. From these values the gas-oil ratio and subsequently the oil recovery may be calculated at various pressures throughout the life of the reservoir by the following equation:

$$R = \frac{Q_g}{Q_o} = R_s + B_o B_g \frac{\mu_o k_g}{\mu_g k_o} \qquad (2)$$

$$\text{Cumulative Recovery} = \left(\frac{S_o}{B_o}\right)_i - \left(\frac{S_o}{B_o}\right) \qquad (3)$$

where the subscript i indicates initial values of S_o and B_o.

From these calculated values, reservoir performance curves may be plotted showing reservoir pressure versus cumulative gas production, reservoir pressure versus oil saturation and reservoir pressure, and producing gas-oil ratio versus recovery of oil in percent or in cumulative barrels.

These curves then may be analyzed for indications of when pressure maintenance programs will be necessary, if additional drilling is necessary, and the best withdrawal rate from the reservoir. Answers to these questions are necessary in order to insure maximum recovery from any reservoir.

In solving equation (1), a Runge-Kutta numerical technique for solving differential equations was employed. Because the input data required for this program is in tabular form, a Lagrange polynomial interpolation method also was used to interpolate between the tabular values. This is not necessary because the data could be expressed in polynomials through the use of regression techniques.

A Note of Caution.- Extreme care should be exercised to insure that the data being used are expressed in the proper units as defined in the symbolic dictionary in the Appendice.

Acknowledgments.- The author is grateful to Floyd W. Preston of The University of Kansas and to Roy Knapp of Northern Natural Gas Company for their assistance and advice. I also express my appreciation to the personnel of the Systems and Data Processing Division of Northern Natural Gas Company, especially Bob Higgs, for their assistance and cooperation in completing the computer program.. Northern Natural Gas Company is also acknowledged for permission to publish this report.

DESCRIPTION OF PROGRAM

The computer program is written in ALGOL for the Burroughs B5500 computer. Pressure-volume-temperature (PVT) and permeability-oil saturation data in tabular form, as well as certain reservoir properties, are required as input to this program. The tabular data are set up in ascending order [i.e., P(1) and $S_o(1)$ must be the highest value in the tables]. A procedure called LAGDUAL is used to perform a Lagrange polynomial interpolation within the tabular data and also to generate the necessary first derivatives of these values. The input forms are shown in the Appendice.

If B_g, the gas formation volume factor, is not known, an option has been written into the program where B_g will be computed from a generalized gas compressibility factor correlation. This correlation is written in a real procedure which requires the gas gravity, the mole fraction of CO_2 and N_2, and the reservoir temperature (degrees Fahrenheit), and pressure (psia), at which the compressibility factor is required.

A Runge-Kutta third order predictor-corrector numerical technique is used to solve the basic equation. The equation is first solved at a value of P_{max} and then the pressure is lowered by successive decrements, ΔP, until either the pre-set abandonment pressure, P_{min}, or the critical oil saturation is reached. The critical oil saturation is that value of S_o at which k_o is zero. The value of ΔP may be any desired value as long as enough points are obtained to permit the definition of the performance curves. Too large a value for ΔP will introduce large errors in the calculations of oil recovery and in the producing gas-oil ratios.

P_{max} and P_{min} must not lie outside the data table and similarly S_{oi} and S_o(critical) must be within the data table.

PROGRAM LIMITATION

The only program limitation as presently written, in addition to the assumptions made in the development of the basic equations, is that the procedure used to compute B_g from a generalized compressibility correlation is not stable at pressures higher than 5000 psia and at gas gravities greater than 0.75. There are, however, procedures available for which this limitation does not apply and which may be inserted in this program with minor revisions. The procedure herein used is based on the AGA Gas Measurement Committee Supercompressibility Report written on PAR Project NX-19.

TESTING OF PROGRAM

Data were taken from an article by Guerrero (1961) in which the Muskat Equation was used to predict the performance of a hypothetical solution gas-drive reservoir. These data were input to the program, and results obtained were compared with those published.

Guerrero took plots of the various data and developed the slopes for several straight line portions. These slopes then were used to compute S_o as a function of p for these ranges. This procedure may be used because generally portions of these curves may be approximated by a straight line.

The program was run with the data using ΔP of 50 psia, 100 psia, 200 psia, 300 psia, 400 psia, and 500 psia. Results obtained from these runs are summarized in the table below, and Guerrero's results are shown for comparison purposes.

The performance curves obtained from some of these runs are shown in the Appendice.

SUGGESTIONS FOR FURTHER PROGRAM DEVELOPMENT

The program has been written for the ideal solution gas drive reservoir with no gas cap and no water encroachment. The program can be expanded to include these terms with several modifications. Hoss (1948) used the expanded Muskat Equation which included a gas cap term. An addition of a water-encroachment term to the material balance equation will eliminate the water problem.

Any of the various steady or unsteady state water influx equations could be easily programmed as a procedure and incorporated in this program.

SUMMARY AND CONCLUSIONS

The program described here will save many man-hours in predicting the performance of solution gas drive reservoirs as well as insure consistency of results. To date, the program has been tested only with data for a hypothetical reservoir, but nevertheless, results were as expected.

By adding a term in the basic equation for water encroachment and a gas cap, and making the necessary modifications to the program, a tool can be created which will greatly simplify the analysis of solution gas drive reservoirs in the early stages of production.

Table 1.- Results of calculations using Muskat Equation in computer program with various ΔP values compared with values obtained by Guerrero (1961).

	Percent recovery to 100 psia	Oil saturation at 100 psia
Guerrero	24.84	.4363
$\Delta P = 50$	24.26	.4397
$\Delta P = 100$	24.25	.4397
$\Delta P = 200$	24.21	.4399
$\Delta P = 300$	24.08	.4407
$\Delta P = 400$	24.05	.4409
$\Delta P = 500$	27.29	.4221

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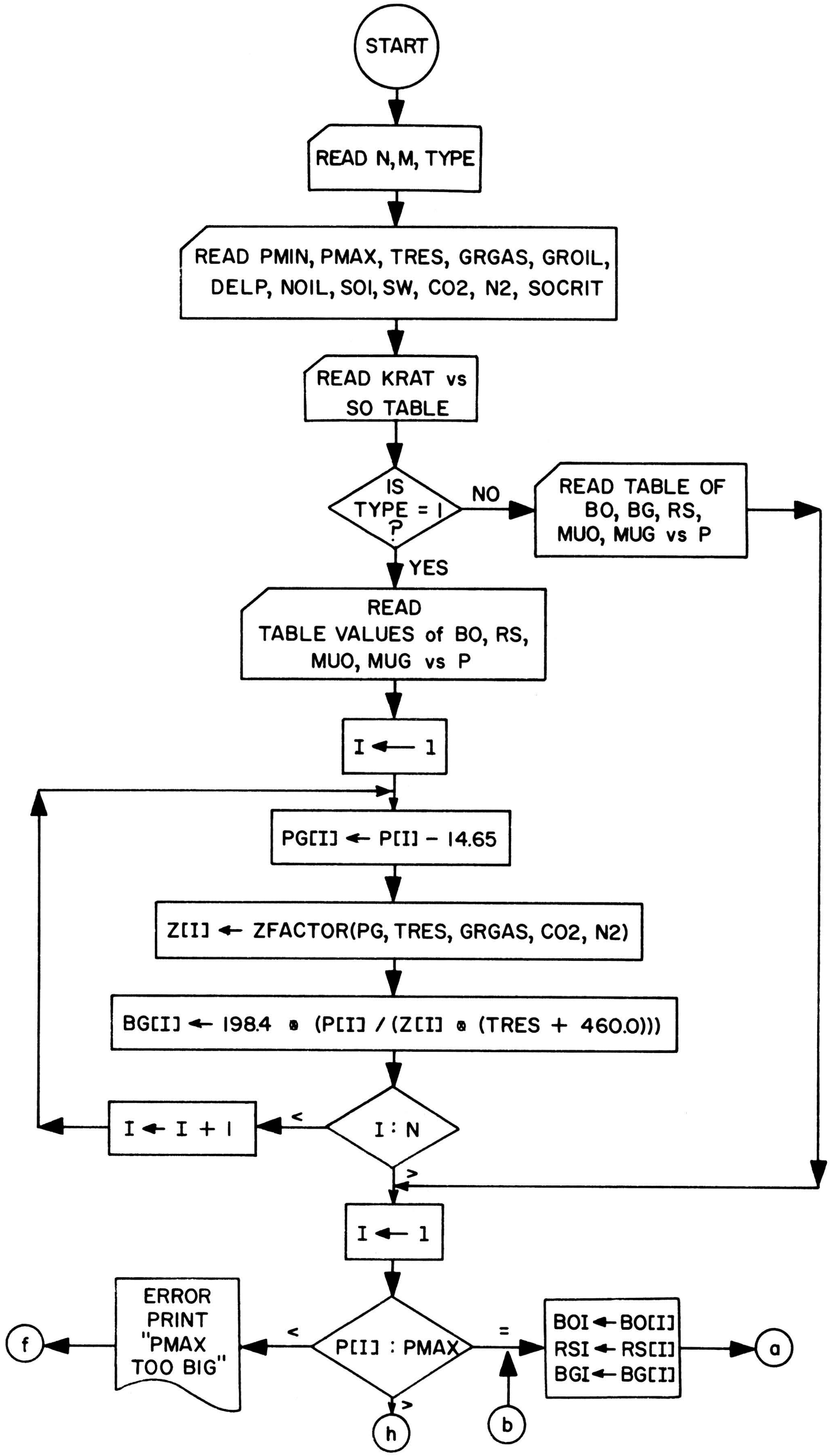
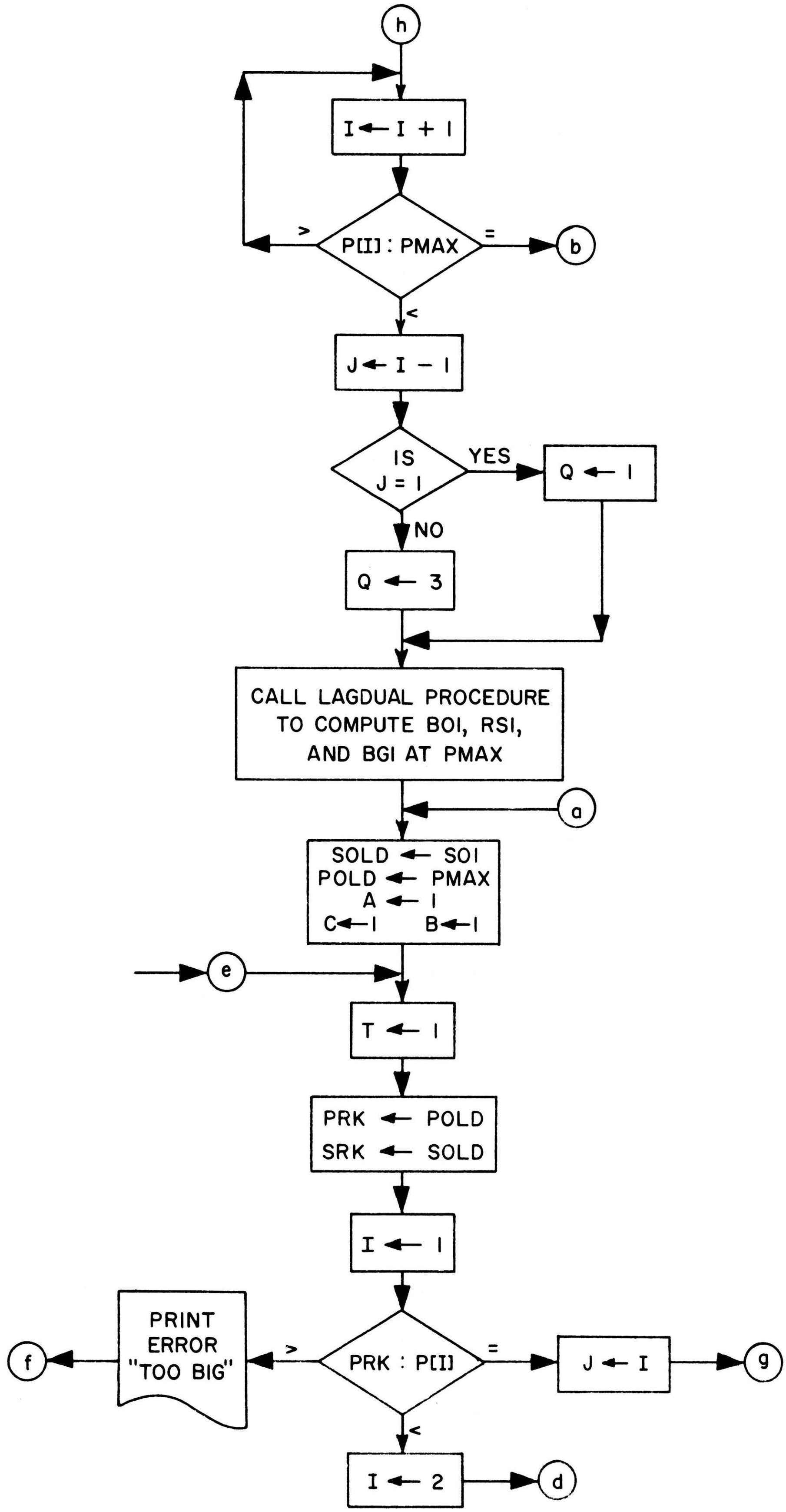
Old, R.E., Jr., 1943, Analysis of reservoir performance: Am. Inst. Mining Eng. Trans., v. 151, p. 86-98.

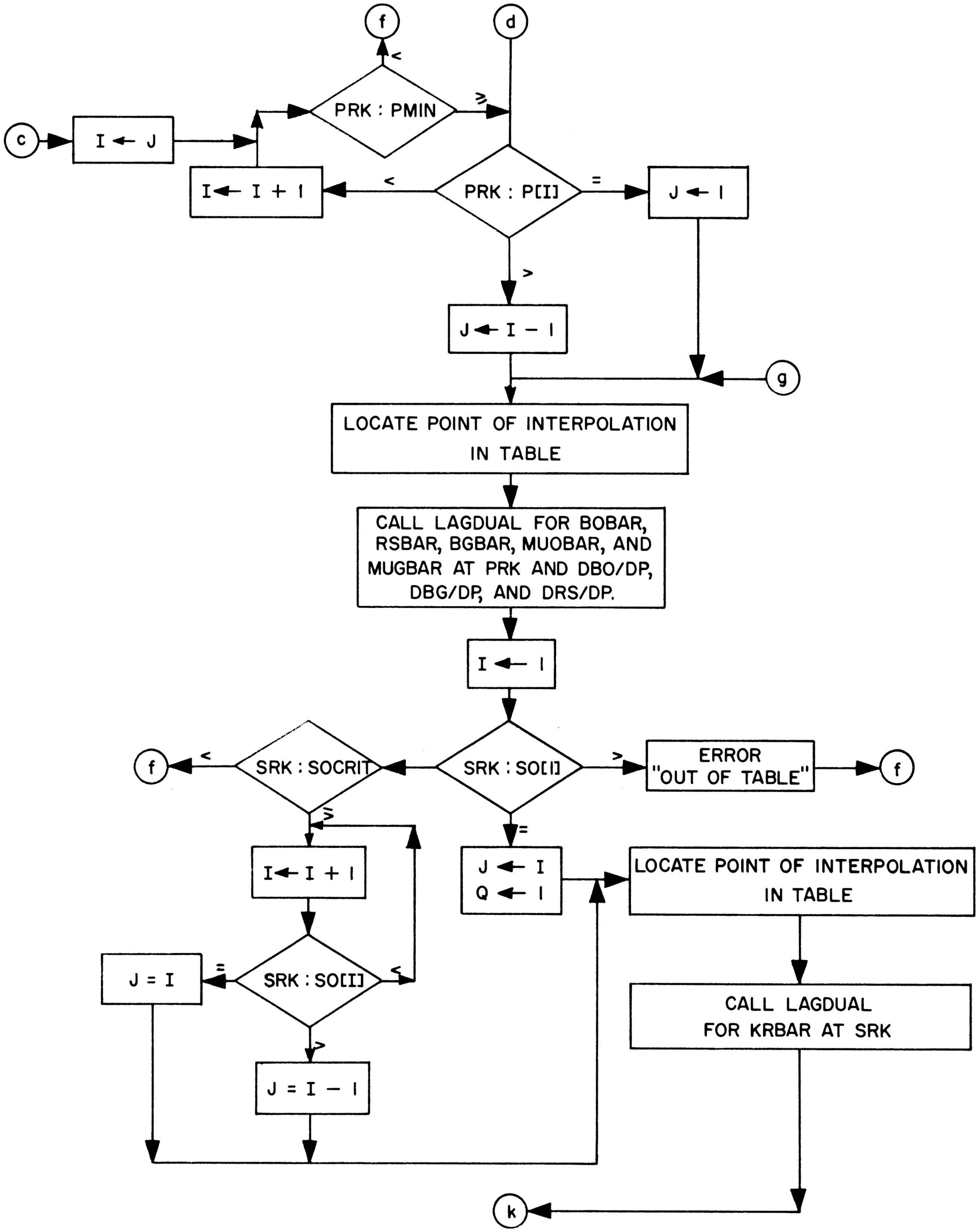
Schilthuis, R.J., 1936, Active oil and reservoir energy: Am. Inst. Mining Eng. Trans., v. 118, p. 33-52.

Tarner, J., 1944, How different size gas caps and pressure maintenance programs affect amount of recoverable oil: Oil Weekly, v. 144, no. 2, p. 32-34.

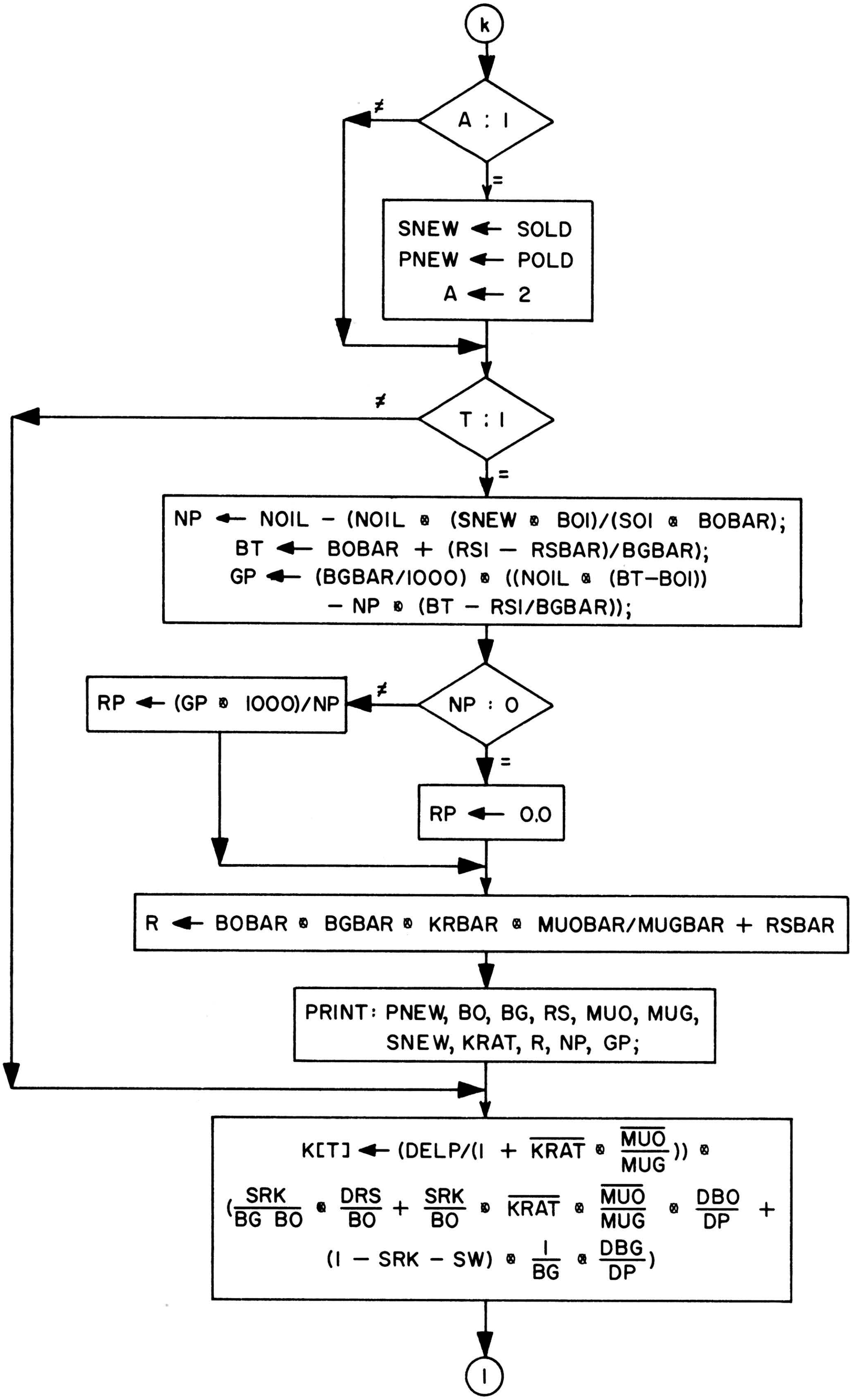
APPENDIX A. - Flow Chart for Muskat Equation Program

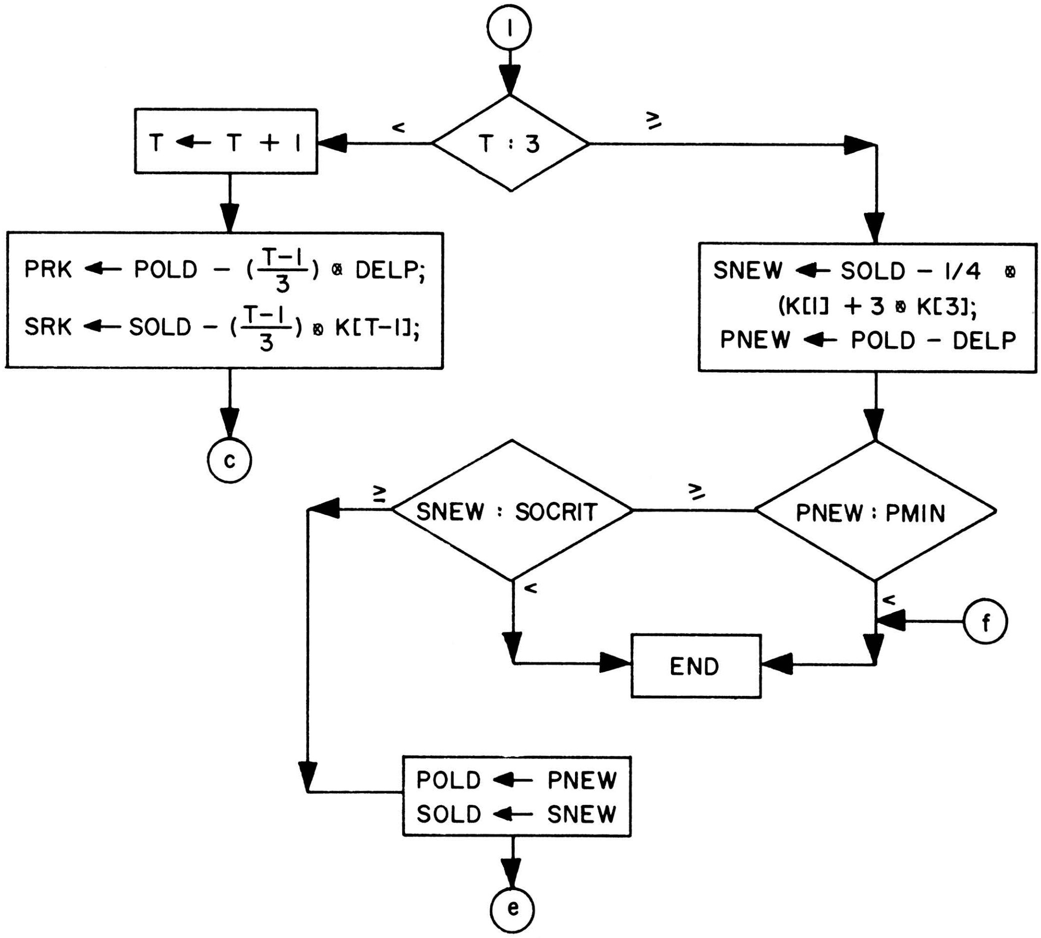



h
I ← I + 1
P[I] : PMAX
>
=
b
<
J ← I − 1
IS
J = 1
YES
Q ← 1
NO
Q ← 3
CALL LAGDUAL PROCEDURE TO COMPUTE BOI, RSI, AND BGI AT PMAX
a
SOLD ← SOI
POLD ← PMAX
A ← 1
C ← 1 B ← 1
e
T ← 1
PRK ← POLD
SRK ← SOLD
I ← 1
PRINT ERROR "TOO BIG"
f
PRK : P[I]
J ← I
g
I ← 2
d



f
d
c
I ← J
PRK : PMIN
<
≥
I ← I + 1
<
PRK : P[I]
=
J ← I
>
J ← I − 1
g
LOCATE POINT OF INTERPOLATION IN TABLE
CALL LAGDUAL FOR BOBAR, RSBAR, BGBAR, MUOBAR, AND MUGBAR AT PRK AND DBO/DP, DBG/DP, AND DRS/DP.
I ← 1
f
<
SRK : SOCRIT
SRK : SO[I]
>
ERROR "OUT OF TABLE"
f
≥
=
I ← I + 1
J ← I
Q ← 1
LOCATE POINT OF INTERPOLATION IN TABLE
J = I
=
SRK : SO[I]
<
CALL LAGDUAL FOR KRBAR AT SRK
>
J = I − 1
k


k
A : I
≠
=
SNEW ← SOLD
PNEW ← POLD
A ← 2
T : I
≠
=
NP ← NOIL − (NOIL ⊗ (SNEW ⊗ BOI)/(SOI ⊗ BOBAR);
BT ← BOBAR + (RSI − RSBAR)/BGBAR);
GP ← (BGBAR/1000) ⊗ ((NOIL ⊗ (BT−BOI)) − NP ⊗ (BT − RSI/BGBAR));
NP : 0
≠
=
RP ← (GP ⊗ 1000)/NP
RP ← 0.0
R ← BOBAR ⊗ BGBAR ⊗ KRBAR ⊗ MUOBAR/MUGBAR + RSBAR
PRINT: PNEW, BO, BG, RS, MUO, MUG, SNEW, KRAT, R, NP, GP;
K[T] ← (DELP/(1 + KRAT ⊗ MUO/MUG)) ⊗ (SRK/BG BO ⊗ DRS/BO + SRK/BO ⊗ KRAT ⊗ MUO/MUG ⊗ DBO/DP + (1 − SRK − SW) ⊗ 1/BG ⊗ DBG/DP)
l


1
T : 3
<
≥
T ← T + 1
PRK ← POLD − ((T−1)/3) ⊗ DELP;
SRK ← SOLD − ((T−1)/3) ⊗ K[T−1];
c
SNEW ← SOLD − 1/4 ⊗
(K[1] + 3 ⊗ K[3];
PNEW ← POLD − DELP
PNEW : PMIN
≥
<
SNEW : SOCRIT
≥
<
f
END
POLD ← PNEW
SOLD ← SNEW
e

APPENDIX B.- Symbolic dictionary for main program.

Algebraic Symbol	ALGOL Symbol	Type	Description and Units
B_g	BG(I)	REAL ARRAY	Gas formation volume factor, SCF/BBL. Input data in tabular form, one value for each pressure. If not available for input, it will be computed using ZFACTOR Procedure.
$\bar{B}_g$	BGBAR	REAL	The desired interpolation in the table of gas formation volume factors for pressure, PRK.
B_{gi}	BGI	REAL	The gas formation volume factor at PMAX. SCF/BBL.
B_o	BO(I)	REAL	Oil formation volume factor BBL/STB. Input data in tabular form, one for each pressure.
$\bar{B}_o$	BOBAR	REAL	The desired interpolation in the oil formation volume factor table for pressure, PRK.
B_{oi}	BOI	REAL	The oil formation volume factor at PMAX. BBL/STB.
B_t	BT	REAL	The two-phase formation volume factor, BBL/STB.
CO_2	CO_2	REAL	Carbon dioxide content of gas, fraction.
dB_g/dP	DBG	REAL	The first derivative of the curve of P vs. B_g. Determined by using Procedure LAGDUAL.
dB_o/dP	DBO	REAL	The first derivative of the curve P vs. B_o. Determined by Procedure LAGDUAL.
dR_s/dP	DRS	REAL	The first derivative of the curve P vs. R_s. Determined by Procedure LAGDUAL.
G	GRGAS	REAL	Gas gravity (air = 1) limited to maximum of 0.75 due to ZFACTOR Procedure, however, if B_g is known no limit is set on gas gravity.
G_o	GROIL	REAL	API gravity of oil.
G_p	GP	REAL	Cumulative gas production, MSCF.
G_r			Remaining gas at any pressure P, SCF. Used in derivation of Muskat's equation.
h			Net pay thickness, feet. Used in Muskat equation derivation.
i	I	INTEGER	A subscript.
K_t	K(T)	REAL ARRAY	An intermediate value of the Runge-Kutta numerical method of solving differential equations.
K_g/K_o	KRAT(I)	REAL ARRAY	Ratio of K_g, permeability to gas, to K_o, permeability to oil. One value for each oil saturation.
$\overline{K_g/K_o}$	KRBAR	REAL	The desired interpolation in the table of permeability ratios for saturation, SRK.
m		REAL	The ratio of initial gas cap volume to initial oil reservoir volume.
N_2	N2	REAL	Nitrogen content of gas, fraction.

N_o	NOIL	REAL	Initial oil in place STB. If 1 bbl. is used, the output is fractional recovery.
N_p	NP	REAL	Cumulative oil produced at any pressure P, STB. If oil in place is 1 bbl., N_p will be fractional.
N_r			Remaining oil at any pressure P, STB. Used in derivation of Muskat's equation.
P	P(I)	REAL ARRAY	Reservoir pressure, psia. Input data in tabular form. Upper limit is 5000 psia due to limits on ZFACTOR Procedure; however, if BG is known and entered as input data there are no upper limits on pressure.
P	PNEW	REAL	The pressure at which the present calculations are done, psia.
P	POLD	REAL	The pressure at which the previous calculations were made.
P	PRK	REAL	The pressure at any time in the reservoir history at which the other variables are being evaluated to solve the Muskat Equation. PRK is used within the Runge-Kutta calculations only.
P_e			The external boundary reservoir pressure, psia. Used in derivation of Muskat's Equation.
P_g	PG(I)	REAL ARRAY	Reservoir pressure, psig. Used to compute Z(I).
P_{max}	PMAX	REAL	The pressure at the start of calculations for the present study, may or may not be the first value in the data table, but must be within the table, psia.
P_{min}	PMIN	REAL	The final pressure which is to be considered for computations. Possibly abandonment pressure. Must be greater than the next to last value of pressure table.
P_w			The bottom hole pressure, psia. Used in derivation of Muskat's Equation.
-	Q	INTE-GER	Order of interpolation.
q_g			Flow rate of gas at mean reservoir temperature and reservoir temperature, bbls/day. Used in Muskat Equation derivation.
q_o			Flow rate of oil bbls/day. Used in derivation of Muskat Equation.
R	R	REAL	Producing gas-oil ratio SCF/STB.
r			Fraction of gas produced which is injected. Used in derivation of expanded Muskat Equation.
r_e			External reservoir boundary radius, feet. Used in derivation of Muskat Equation.
R_p	RP	REAL	Cumulative gas-oil ratio SCF/STB.
R_s	RS(I)	REAL	Solution gas-oil ratio, SCF/STB. Input data in tabular form, one for each pressure.
$\bar{R}_s$	RSBAR	REAL	The desired interpolation in the solution gas-oil ratio table for a pressure, PRK.
R_{si}	RSI	REAL	The value of solution gas-oil ratio at PMAX, SCF/STB.

r_w			The wellbore radius, feet. Used in derivation of Muskat's Equation.
ΔP	DELP	REAL	The desired increment of pressure for solving the Muskat Equation.
μ_g	MUG(I)	REAL ARRAY	Gas viscosity centipoise. Input data in tabular form, one for each pressure.
$\overline{\mu_g}$	MUGBAR	REAL	The desired interpolation in the table of gas viscosities for a pressure PRK.
μ_o	MUO(I)	REAL	Oil viscosity in centipoise. Input data in tabular form, one for each pressure.
$\overline{\mu_o}$	MUOBAR	REAL	The desired interpolation in the oil viscosity table for a pressure, PRK.
	A	INTEGER	A counter to tell when the calculations are made at P_{max} and S_{oi}.
	DK	REAL	A value used only to complete the Procedure LAGDUAL call statement when interpolating for values of K_g/K_o.
	DMG	REAL	Used only to complete the call statement for Procedure LAGDUAL when interpolating for μ_g .
	DMO	REAL	Used only to complete the argument in Procedure LAGDUAL when interpolating for μ_o.
	M	INTEGER	The number of values in the K_g/K_o versus oil saturation table.
	N	INTEGER	The number of values in the pressure table.
S_o	SNEW	REAL	The oil saturation at PNEW.
S_o	SO(I)	REAL	A table of values read in with values of K_g/K_o. The table should include the value of the initial oil saturation at PMAX or SOI.
S_o	SOLD	REAL	The previously computed value of oil saturation, or the value at POLD.
$\overline{S_o}$	SRK	REAL	The intermediate value of oil saturation which is computed within the Runge-Kutta portion of the program.
$S_{o\,(crit)}$	SOCRIT	REAL	The value of S_o at which K_o becomes zero. This value must be greater than the next to last value in the saturation table.
S_{oi}	SOI	REAL	Oil saturation at PMAX.
S_w	SW	REAL	Connate water saturation fraction.
t	T	INTEGER	The subscript of K used in Runge-Kutta calculations.
T_r	TRES	REAL	Reservoir temperature in (^oF).
TYPE		INTEGER	An integer to indicate whether BG(I) is to be read in or computed within the program. If TYPE = 1, BG will be computed; otherwise, it must be input.
V_p			Reservoir pore volume, barrels. Used in derivation of Muskat Equation.
Z	Z(I)	REAL ARRAY	Gas compressibility factor, computed from Procedure ZFACTOR and used to compute B_g when not available for input.

APPENDIX C. - Listing of ALGOL program.

```
        BEGIN                                                                   0000
                                                          START OF SEGMENT **********     2
COMMENT         THIS IS A PROGRAM TO COMPUTE THE PERFORMANCE OF A               0000
                SOLUTION GAS DRIVE RESERVOIR AT OR BELOW ITS BUBBLE             0000
                POINT PRESSURE BY USING THE MUSKAT EQUATION AND SOLVING         0000
                IT BY THE RUNGE-KUTTA NUMERICAL DIFFERENTIATION METHOD          0000
                          PROGRAMMED BY A. BACA 5/7/64;                         0000
FILE IN         CARDS   (1,10);                                                 0000
FILE OUT        PRINT 4 (1,15);                                                 0003
REAL            AOS;                                                            0007
INTEGER       A,N,M,I,J,T,Q,TYPE,B,C,R         ;                                0007
        BEGIN                                                                   0007
LABEL           EXIT;                                                           0007
                                                          START OF SEGMENT **********     3
LIST            SIZE(N,M,TYPE);                                                 0000
FORMAT IN      FS1(I4,I4,I4);                                                   0008
                                                          START OF SEGMENT **********     4
                                                             4 IS    6 LONG, NEXT SEG     3
                READ(CARDS,FS1,SIZE);                                           0008
        BEGIN                                                                   0012
REAL ARRAY      P,BO,RS,MUO,MUG,Z,BG,PG [1:N],KRAT,SO[1:M],K[1:3];              0012
                                                          START OF SEGMENT **********     5
REAL            DBO,DBG,DRS,DMO,DMG,BOI,BGI,RSI,DK,KRBAR,BOBAR,BGBAR, R,        0010
                BT,SW,MUGBAR,RSBAR,PRK,SRK,PMAX,PMIN,SOI,SOLD,POLD,DELP,        0010
                MUOBAR,PNEW,SNEW,   SOCRIT,NP,NOIL,GP,RP,GRGAS,CO2,N2,          0010
                GROIL,TRES;                                                     0010
LABEL           E1,SAME,L1,SAME1,RKS, L7, COM,RKR,LAG1,L8,L10, L11      ;      0010
LABEL           COM2,LAG2       ;                                               0010
LIST            LP(PNEW,BOBAR,BGBAR,RSBAR,R ,SNEW,KRBAR,MUOBAR,MUGBAR,NP,       0010
                GP);                                                            0024
LIST            LL(  PMAX,PMIN,SOI,SW,SOCRIT,NOIL,DELP     ));                  0029
LIST            L1A(GRGAS,GROIL,CO2,N2,TRES);                                   0042
LIST            LZN(FOR I ← 1 STEP 1 UNTIL N DO[P[I],BO[I],BG[I],RS[I],         0052
                MUO[I],MUG[I]]);                                                0063
LIST            L2(FOR I ← 1 STEP 1 UNTIL N DO [P[I],BO[I],RS[I],MUO[I],        0072
                MUG[I]]);                                                       0083
LIST            L3(FOR I ← 1 STEP 1 UNTIL M DO [SO[I],KRAT[I]]);                0090
FORMAT IN       F1(   X5,F8.1,X1,F8.1,3(X1,F7.4),X1,F11.1,X1,F6.1,X5,           0102
                                                          START OF SEGMENT **********     6
                X1      ),                                                      0102
                F1A(F7.4,X3,F5.1,X3,F5.2,X3,F5.2,X3,F6.1),                      0102
                FS(F7.4,X3,F8.4),                                               0102
                FZA(F9.1,X3,F7.3,X3,F9.1,X3,F9.1,X3,F8.2,X2,F7.4),              0102
                FP(F9.1,X3,F7.3,X3,F9.1,X3,F8.2,X2,F7.4));                      0102
                                                             6 IS   61 LONG, NEXT SEG     5
FORMAT OUT      ER("PMAX TOO BIG OR OUT OF TABLE"),                             0102
                                                          START OF SEGMENT **********     7
                HDG1(X46,"NORTHERN  NATURAL  GAS  CO.",//,X46,"KANSAS  ",       0102
                "UNIVERSITY  RESEARCH",//,X56,"PROJECT",///,X43,"PERFOR",       0102
                "MANCE  OF  A  SOLUTION  GAS",//,X52,"DRIVE RESERVOIR",//       0102
                ,X49,"BY  MUSKAT S  EQUATION",//,"  PRES    OIL FVF   G",       0102
                "AS FVF   SOL GOR    PRO  GOR  OIL        KG/KO       VIS",     0102
                "COS   VISCOS         CUM OIL          CUM GAS",/," PSIA  ",    0102
                "  BBL/STB    SCF/BBL   SCF/STB      SCF/STB  SATUR      ",     0102
                "            OIL CP    GAS CP        BBLS            MCF   ")   0102
                ;                                                               0102
                                                             7 IS   97 LONG, NEXT SEG     5
FORMAT OUT      ANS(I7,X3,F6.3,X3,3(I7,X3),F7.4,X3,F8.4,X3,F8.2,X2,F7.4,        0102
                                                          START OF SEGMENT **********     8
                X3,E14.7,X2,E14.7));                                            0102
                                                             8 IS   22 LONG, NEXT SEG     5
COMMENT         INSERT PROCEDURE ZFACTOR HERE;                                  0102
COMMENT         INSERT PROCEDURE LAGDUAL HERE;                                  0102
REAL PROCEDURE ZFACTOR   (PRESS,TEMP,GR,CO2,N2);                                0102
VALUE           PRESS,TEMP,GR,CO2,N2;                                           0102
REAL            PRESS,TEMP,GR,CO2,N2;                                           0102
        BEGIN                                                                   0102
REAL            PI,TAU,K1,K2,K3,E,PI2,K4,Y1,M1,KZ,B1 ,N1,D1;                    0102
                                                          START OF SEGMENT **********     9
DEFINE          E1 =1-K1×EXP(-20×K2)- .0011×K2×.5×PI2×(2.17 + 1.4 × K2          0000
                  * .5 -PI) *2  #,                                              0000
                E3 =1-K1×(2-EXP(-20×K3))+ .455×(200×K3*6+K3×(-.03249+           0000
                    K3×(2.0167 +K3×(-18.028 +K3 ×42.844)))×(PI-1.3)             0000
                    ×(4.019496-PI2)) #;                                         0000
```

```
                C02←100×C02;                                              0000
                N2←N2×100;                                                0001
                PI←(0.15647×PRESS) / (160.8 - 7.22 × GR +C02 - .392 × N2) 0002
                   +  0.0147;                                             0005
                TAU← ( .45258 × (TEMP + 460)) / (99.15 + 211.9× GR- C02   0007
                    -1.681× N2) ;                                         0009
                K1 ←  .00075 × PI * 2.3;                                  0012
                K2 ←  TAU - 1.09;                                         0015
                K3 ← -K2;                                                 0016
                PI2←PI×PI;                                                0017
                IF PI≥0 AND PI≤2 AND TAU≥1.09 AND TAU≤1.4 THEN E←E1 ELSE  0018
                IF PI≥0 AND PI≤1.3 AND TAU≥.84 AND TAU≤1.09 THEN          0032
                E←1-K1×(2-EXP(-20×K3))-1.317×K3*4×PI×(1.69-PI2)ELSE       0052
                IF PI≥1.3 AND PI≤2 AND TAU≥.88 AND TAU≤1.09 THEN E←E3     0060
                ELSE IF PI>2 AND PI≤5 THEN                                0082
            BEGIN                                                         0094
                K4←PI - 2.0;                                              0095
                Y1←K4×((1.7172+TAU×(-2.33123+TAU×(-1.56796+TAU×(3.47644   0096
                +TAU×(-1.28603))))) + K4 × ((.016299+TAU×(-.028094+TAU×   0098
                (.48782+TAU×(-.728221+TAUx .27839)))) + K4 ×((-0.35978    0103
                +TAU×(0.51419 +TAU ×(0.16453+TAU×(-0.52216 + TAU ×        0107
                0.19687)))) +K4 ×(0.075255+TAU×(-0.10573 + TAU ×          0109
                (-0.058598+TAU×(0.14416 + TAU ×(-0.054533))))))));        0113
                E← IF TAU≥.88 AND TAU≤1.09 THEN  E3-Y1                    0119
                ELSE IF TAU>1.09 AND TAU≤1.32 THEN E1-Y1                  0134
                ELSE IF TAU>1.32 AND TAU≤1.4 THEN E1-Y1-(K4×(TAU-1.32)*2  0146
                ×(3.0 +K4×(-1.483 + K4×(-.10 +K4 × 0.0833))))ELSE 1 ;     0160
            END ELSE E←1;                                                 0166
                K1← 1 / (TAU×TAU);KZ←1/TAU;                               0206
                M1← 0.0330378 × K1 -0.0221323 ×K1×KZ  + 0.0161353×K1×K1   0209
                   ×KZ;                                                   0212
                N1← (0.265827 ×K1+ 0.0457697×K1×K1-0.1331850×KZ)/ M1;     0214
                B1←(3 -M1×N1*2) /(9 ×M1×PI2);                             0218
                K2←(9 ×N1-2 ×M1 ×N1*3) / (54 × M1 ×PI×PI2)-E/(2×M1×PI2);  0222
                D1←(K2+(K2*2+B1*3)*.5) *0.333333;                         0230
                ZFACTOR ←(1 +(.00132 / (TAU*3.25)))*2 /                   0237
                      (B1/ D1-D1+N1/(3×PI)) ;                             0241
            END ;                                                         0244
                                                9 IS  261 LONG, NEXT SEG      5
PROCEDURE       LAGDUAL(Q,J,X,Y,XBAR,YBAR,DY,SW,NUMB);                    0102
VALUE           Q,J,X,Y,XBAR,SW,NUMB;                                     0102
INTEGER         J,Q,SW,NUMB;                                              0102
REAL ARRAY      X,Y[1];                                                   0102
REAL                XBAR,YBAR,DY;                                         0102
            BEGIN                                                         0102
COMMENT         THIS PROCDURE IS USED TO DO POLYNOMIAL INTERPOLATION WITH 0102
                THE  LAGRANGE EQUATION  AND BY SETTING SW TO 1 THE        0102
                FIRST DERIVATIVE OF THE DEPENDENT VARIABLE WITH RESPECT   0102
                TO THE INDEPENDENT VARIABLE CAN ALSO BE OBTAINED;         0102
REAL            DENOM,NUM,P,SP;                                           0102
                                            START OF SEGMENT **********      10
INTEGER         L,I,R,K,LL;                                               0000
LABEL           EVEN,ST,L1,L2,L3,L4,CHK,CHK1,INT,OT;                      0000
                IF Q -(2 × ENTIER(Q/2)) = 0 THEN GO TO EVEN ELSE I ← J -  0000
                Q DIV 2;                                                  0004
                GO TO  ST;                                                0005
                                                                          0007
EVEN:           IF (X[J] - XBAR)  ≤ ((X[J] - X[J+1])/2) THEN I ← J - Q    0007
                DIV 2 ELSE I ← J - (Q DIV 2 -1);                          0012
ST:             YBAR ← 0.0;                                               0017
                K ← I + Q ;                                               0018
                IF I = NUMB THEN  K ← I;                                  0019
                DY ← 0.0;                                                 0021
                R ← I;                                                    0022
L1:                                                                       0023
                NUM ← 1.0;                                                0023
                DENOM ← 1.0;                                              0023
                P ← 0.0;                                                  0024
                SP ← 0.0;                                                 0025
                L ← I;                                                    0026
L2:             IF L ≠ R THEN GO TO CHK;                                  0026
                GO TO CHK1;                                               0028
CHK:            IF SW ≠ 1 THEN GO TO INT;                                 0030
```

```
            LL ← 1;                                                        LD000310    0031
            P ← 1.0;                                                       LD000320    0032
L4:         IF LL = L THEN GO TO L3 ELSE                                   LD000330    0032
            IF LL = R THEN GO TO L3 ELSE                                   LD000340    0033
            P ← P × (XBAR - X[LL]);                                        LD000350    0035
L3:         IF LL < K THEN                                                 LD000370    0038
        BEGIN                                                              LD000380    0038
            LL ← LL + 1; GO TO L4;                                         LD000390    0039
        END;                                                               LD000400    0042
            SP ← SP + P;                                                   LD000410    0042
INT:        DENOM ← DENOM × (X[R] - X[L]);                                 LD000420    0043
            NUM ← NUM × (XBAR - X[L]);                                     LD000430    0047
CHK1:       IF L < K THEN                                                  LD000440    0049
        BEGIN                                                              LD000450    0050
            L ← L +1; GO TO L2;                                            LD000460    0051
        END;                                                               LD000470    0053
            YBAR ← YBAR + (NUM/DENOM) × Y[R];                              LD000480    0053
            IF SW = 1 THEN DY ← DY + (SP/DENOM) × Y[R] ;                   LD000490    0056
            IF R ≥ K THEN GO TO OT  ELSE R ← R +1;                         LD000500    0060
            GO TO L1;                                                      LD000510    0063
OT:     END;                                                               LD000520    0063
                                                                 10 IS   69 LONG, NEXT SEG    5
            WRITE(PRINT,HDG1);                                             AB000520    0102
            READ(CARDS,F1,LL);                                             AB000530    0105
            READ(CARDS,F1A,L1A);                                           AB000540    0109
            READ(CARDS,FS,L3);                                             AB000550    0113
            IF TYPE ≠ 1 THEN GO TO L7;                                     AB000560    0116
            READ(CARDS,FP,L2);                                             AB000570    0118
            FOR I ← 1 STEP 1 UNTIL N DO                                    AB000580    0121
        BEGIN                                                              AB000590    0123
            PG[I] ← P[I] - 15 ;                                            AB000600    0123
            Z[I]←ZFACTOR(P[I],TRES,GRGAS,CO2,N2);                          AB000610    0126
            BG[I] ←  (198.4 × P[I])/(Z[I] × (TRES + 460.0));               AB000620    0130
        END;                                                               AB000630    0135
            IF TYPE = 1 THEN GO TO L8;                                     AB000640    0137
L7:         READ(CARDS,FZA,LZN);                                           AB000650    0138
L8:         IF P[1] < PMAX THEN GO TO E1 ELSE IF P[1] = PMAX THEN          AB000660    0142
            GO TO SAME1 ELSE I ← 1;                                        AB000670    0146
L1:         I ← I +1;                                                      AB000680    0147
            IF P[I] = PMAX THEN GO TO SAME ELSE IF P[I] > PMAX THEN        AB000690    0149
            GO TO L1 ELSE  J ← I-1;                                        AB000700    0152
            IF J = 1 THEN Q ← 1 ELSE Q ← 3;                                AB000710    0154
            LAGDUAL(Q,J,P,BO,PMAX,BOI,DBO,2,N);                            AB000720    0159
            LAGDUAL(Q,J,P,RS,PMAX,RSI,DRS,2,N);                            AB000730    0163
            LAGDUAL(Q,J,P,BG,PMAX,BGI,DBG,2,N);                            AB000740    0167
            GO TO RKS;                                                     AB000750    0171
SAME1:      BOI ← BO[1];                                                   AB000760    0171
            BGI ← BG[1];                                                   AB000770    0173
            RSI ← RS[1];                                                   AB000780    0175
            GO TO RKS;                                                     AB000790    0176
SAME:       BOI ← BO[I];                                                   AB000800    0177
            BGI ← BG[I];                                                   AB000810    0178
            RSI ← RS[I];                                                   AB000820    0180
COMMENT     THIS IS ENTRANCE TO RUNGE-KUTTA CALCULATIONS;                  AB000830    0181
RKS:        SOLD ← SOI;                                                    AB000840    0181
            POLD ← PMAX;                                                   AB000850    0182
            A ← 1;                     B←C←1;                              AB000860    0183
RKR:        T ← 1;                                                         AB000870    0185
            PRK ← POLD;                                                    AB000880    0186
            SRK ← SOLD;                                                    AB000890    0187
            I ← 1;                                                         AB000900    0188
            IF PRK > P[I] THEN GO TO E1 ELSE IF PRK = P[I] THEN            AB000910    0189
        BEGIN                                                              AB000920    0192
            J ← I; GO TO LAG1;                                             AB000930    0193
        END;                                                               AB000940    0194
            I ← 2;                                                         AB000950    0194
COM:    IF PRK < PMIN THEN GO TO EXIT;                                     AB000960    0195
        IF PRK < P[I] THEN                                                 AB000970    0198
        BEGIN                                                              AB000980    0200
            I ← I +1;                                                      AB000990    0200
            GO TO COM ;                                                    AB001000    0201
        END                                                                AB001010    0202
            ELSE IF PRK = P[I] THEN J ← I ELSE J ← I - 1;                  AB001020    0202
```

```
LAG1:           IF J = 1 THEN Q ← 1 ELSE Q ← 3;                               AB001030    0207
                IF J+Q DIV 2 ≥ N-1 THEN Q←1;                                  AB001040    0211
COMMENT         LAGDUAL PROCEDURE IS CALLED HERE FOR INTERPOLATED VALUES      AB001050    0214
                AND THE REQUIRED DERIVATIVES;                                 AB001060    0214
                LAGDUAL(Q,J,P,BO,PRK,BOBAR,DBO,1,N);                          AB001070    0214
                DBO ← DBO ;                                                               0218
                LAGDUAL(Q,J,P,BG,PRK,BGBAR,DBG,1,N);                          AB001080    0219
                DBG ← DBG ;                                                               0223
                LAGDUAL(Q,J,P,RS,PRK,RSBAR,DRS,1,N);                          AB001090    0223
                DRS ← DRS ;                                                               0227
                LAGDUAL(Q,J,P,MUO,PRK,MUOBAR,DMO,2,N);                        AB001100    0228
                LAGDUAL(Q,J,P,MUG,PRK,MUGBAR,DMG,2,N);                        AB001110    0232
                I←1     ;                                                     AB001120    0235
                IF SRK > SO[I] THEN GO TO E1 ELSE IF SRK = SO[I] THEN         AB001130    0236
BEGIN Q←1;         J ← I; GO TO LAG2; END;        I←2;                        AB001140    0240
COM2:      IF SRK < SOCRIT THEN GO TO EXIT;                                   AB001150    0243
                IF I = M THEN BEGIN I ← I - 1;              J ← I;            AB001160    0247
                GO TO LAG2; END;                                              AB001170    0250
           IF SRK < SO[I] THEN BEGIN I ← I + 1;GO TO COM2; END ELSE           AB001180    0251
                IF SRK = SO[I] THEN J ← I ELSE J ← I - 1;                     AB001190    0255
LAG2:           IF J ≤3 THEN Q ← 1 ELSE Q ← 3;                                AB001200    0260
                IF J+Q DIV 2 ≥ M-1 THEN Q←1;                                  AB001210    0263
                LAGDUAL(Q,J,SO,KRAT,SRK,KRBAR,DK,2,M);                        AB001220    0266
                IF A = 1 THEN                                                 AB001230    0270
           BEGIN                                                              AB001240    0271
                SNEW←SOLD;                                                    AB001250    0271
                PNEW←POLD;                                                    AB001260    0272
                A ← 2 ;                                                       AB001270    0273
           END;                                                               AB001280    0274
                IF T ≠ 1  THEN GO TO L10;                                     AB001290    0274
                NP ←  NOIL  - (NOIL ×  (SNEW × BOI)/(SOI × BOBAR));           AB001300    0275
                BT ← (BOBAR + ((RSI - RSBAR)/BGBAR ));                        AB001310    0278
                GP ←      (BGBAR/1000)    ×      (NOIL ×   ((BT - BOI)        AB001320    0280
                                             ) -NP×(BT-RSI /                  AB001330    0282
                BGBAR) );                                                     AB001340    0283
                IF NP=0 THEN BEGIN RP←0; GO TO L11; END;                      AB001350    0285
                RP ← GP / NP ×1000;                                           AB001360    0288
L11:            R←BOBAR×BGBAR×KRBAR×MUOBAR / MUGBAR +RSBAR ;                  AB001370    0289
                WRITE(PRINT,ANS,LP);                                          AB001380    0293
L10:            K[T] ←    (DELP/(1 + KRBAR × (MUOBAR/MUGBAR)))   × ((( SRK/   AB001390    0296
                (BGBAR × BOBAR)) × DRS) +    SRK/BOBAR × KRBAR × DBO ×        AB001400    0300
                MUOBAR/MUGBAR   + (DBG/BGBAR) ×                               AB001410    0303
                        (1- SW - SRK));                                       AB001420    0305
                AOS←K[T];                                                                 0308
                IF T < 3 THEN                                                 AB001430    0309
           BEGIN                                                              AB001440    0310
                T ← T + 1;                                                    AB001450    0311
                PRK ← POLD  - (((T - 1)/3) × DELP);                           AB001460    0312
                SRK ← SOLD  - (((T -1)/3) × K[T-1]);                          AB001470    0315
                I ← 1;                                                        AB001480    0319
                GO TO COM;                                                    AB001490    0319
           END;                                                               AB001500    0320
                SNEW ← SOLD - ((1/4) × ((K[1]+3×K[3])));                      AB001510    0320
                PNEW ← POLD - DELP;                                           AB001520    0325
                IF B ≠ 1 THEN GO TO EXIT;                                     AB001530    0326
                IF PNEW ≤ PMIN THEN BEGIN PNEW ← PMIN; B ← 2; END;            AB001540    0329
                IF C ≠ 1 THEN GO TO EXIT;                                     AB001550    0332
                IF SNEW ≤ SOCRIT THEN BEGIN SNEW ← SOCRIT; C ← 2; END;        AB001560    0336
                POLD ← PNEW;                                                  AB001570    0338
                SOLD ← SNEW;                                                  AB001580    0339
                GO TO RKR;                                                    AB001590    0340
E1:             WRITE(PRINT,ER);                                              AB001600    0340
           END;                                                               AB001610    0344
                                                                     5 IS  347 LONG, NEXT SEG    3
EXIT:      END;                                                               AB001620    0013
                                                                     3 IS   17 LONG, NEXT SEG    2
           END.                                                               AB001630    0008
                                                                     2 IS   11 LONG, NEXT SEG    1
```

```
      EXP IS SEGMENT NUMBER 0011,PRT ADDRESS IS 0145
      LN IS SEGMENT NUMBER 0012,PRT ADDRESS IS 0144
      OUTPUT(W) IS SEGMENT NUMBER 0013,PRT ADDRESS IS 0150
      BLOCK CONTROL IS SEGMENT NUMBER 0014,PRT ADDRESS IS 0005
      INPUT(W) IS SEGMENT NUMBER 0015,PRT ADDRESS IS 0047
      X TO THE I IS SEGMENT NUMBER 0016,PRT ADDRESS IS 0146
      GO TO SOLVER  IS SEGMENT NUMBER 0017,PRT ADDRESS IS 0152
      ALGOL WRITE   IS SEGMENT NUMBER 0018,PRT ADDRESS IS 0014
      ALGOL READ    IS SEGMENT NUMBER 0019,PRT ADDRESS IS 0015
      ALGOL SELECT  IS SEGMENT NUMBER 0020,PRT ADDRESS IS 0016
                                                                                   1 IS     2 LONG, NEXT SEG    0
                                                                                  21 IS    69 LONG, NEXT SEG    0
NUMBER OF ERRORS DETECTED =    0.  COMPILATION TIME =   77 SECONDS.

PRT SIZE = 109; TOTAL SEGMENT SIZE =   962 WORDS; DISK SIZE =  46 SEGS; NO. PGM. SEGS =  21

ESTIMATED CORE STORAGE REQUIREMENT =  6724 WORDS.
```

APPENDIX D.- Input Forms

FIRST DATA CARD

N	M	TYPE
1 – 4	5 – 8	9 – 12

SECOND DATA CARD

	PMAX PSIA		PMIN PSIA		SOI		SW		SOCRIT		NOIL STB		DELP PSIA
X	6 – 13	X	15 – 22	X	24 – 30	X	32 – 38	X	40 – 46	X	48 – 58	X	60 – 65

THIRD DATA CARD

GRGAS		GROIL		CO_2 MOLE FR		N_2 MOLE FR		TRES °F
1 – 7	X	11 – 15	X	19 – 23	X	27 – 31	X	35 – 40

NEXT "M" DATA CARDS OIL
SATURATION-PERMEABILITY DATA TABLE

SO		KG/KO
1 – 7		11 – 18

NEXT "N" DATA CARDS
PRESSURE DEPENDENT DATA TABLE

P PSIA		BO BBL/STB		BG SCF/BBL		RS SCF/STB		MUO CPS		MUG CPS
1 – 9		13 – 19		23 – 31		35 – 43		47 – 54		57 – 63

USE THIS FORM WHEN
GAS FORMATION VOLUME
FACTOR BG IS NOT AVAILABLE
IN TABULAR FORM.

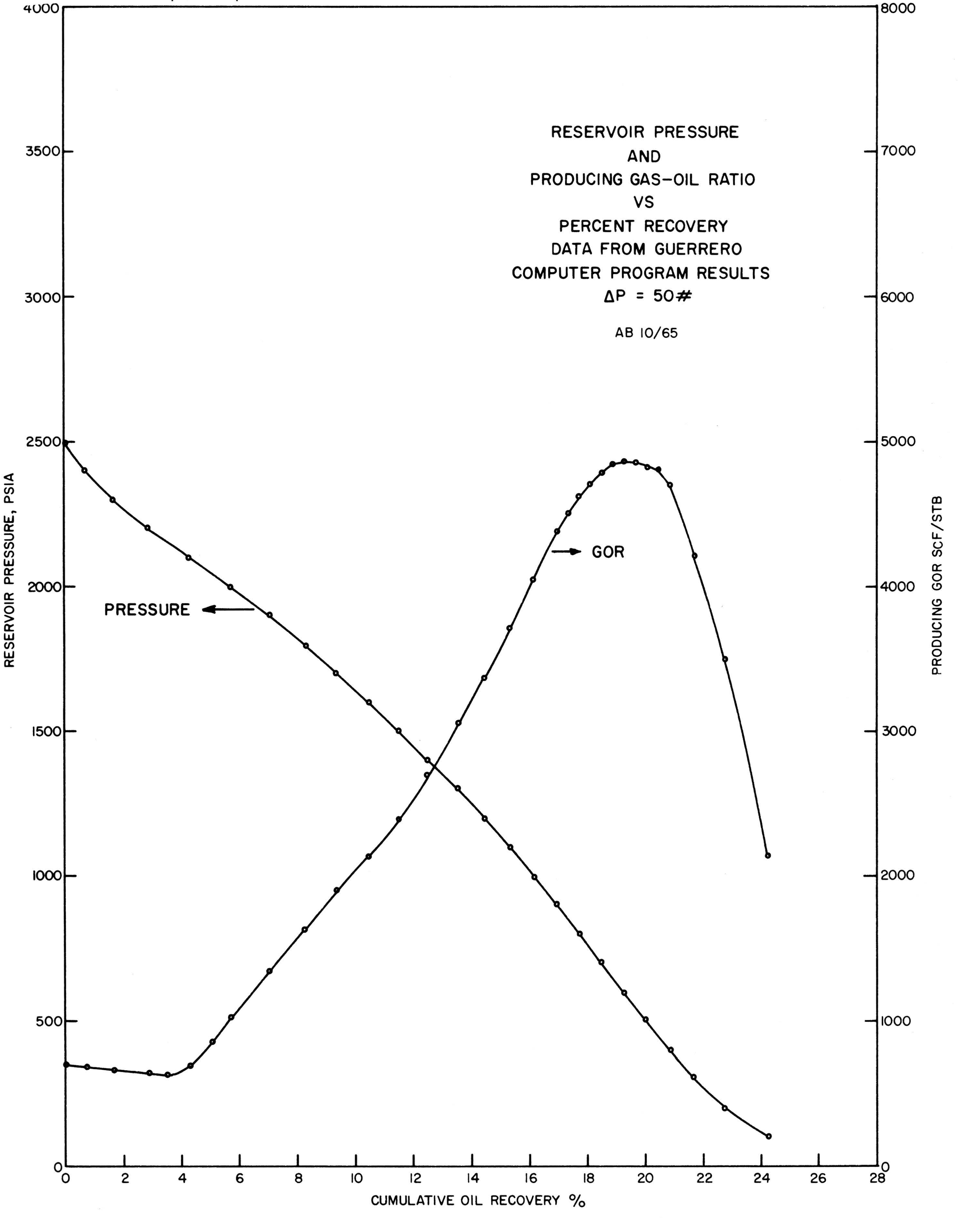
(ALTERNATE)

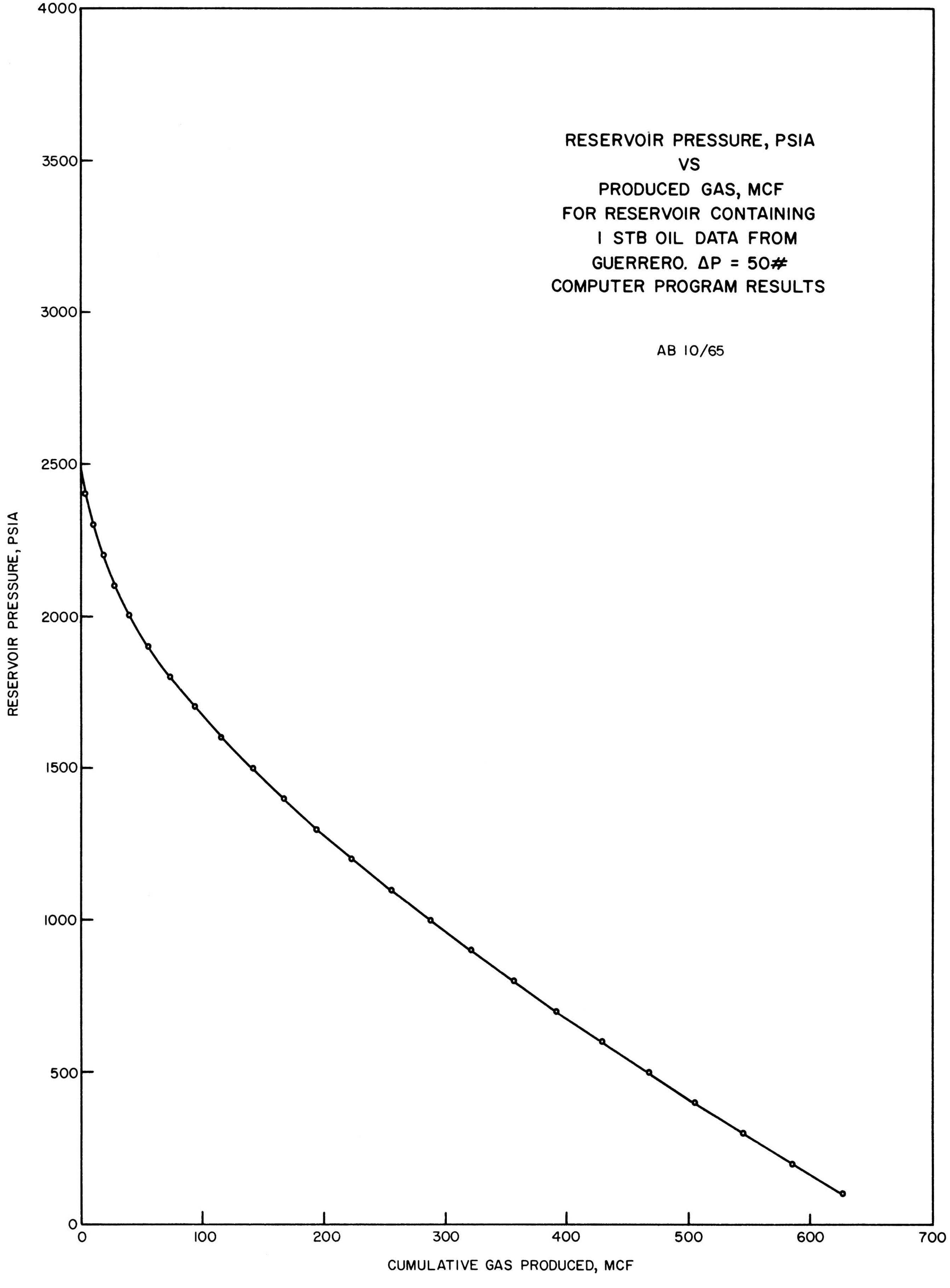
P PSIA		BO BBL/STB		RS SCF/BBL		MUO CPS		MUG CPS
1 – 9		13 – 19		23 – 31		35 – 42		45 – 51

Input data for performance of a solution gas drive reservoir by Muskat's Equation

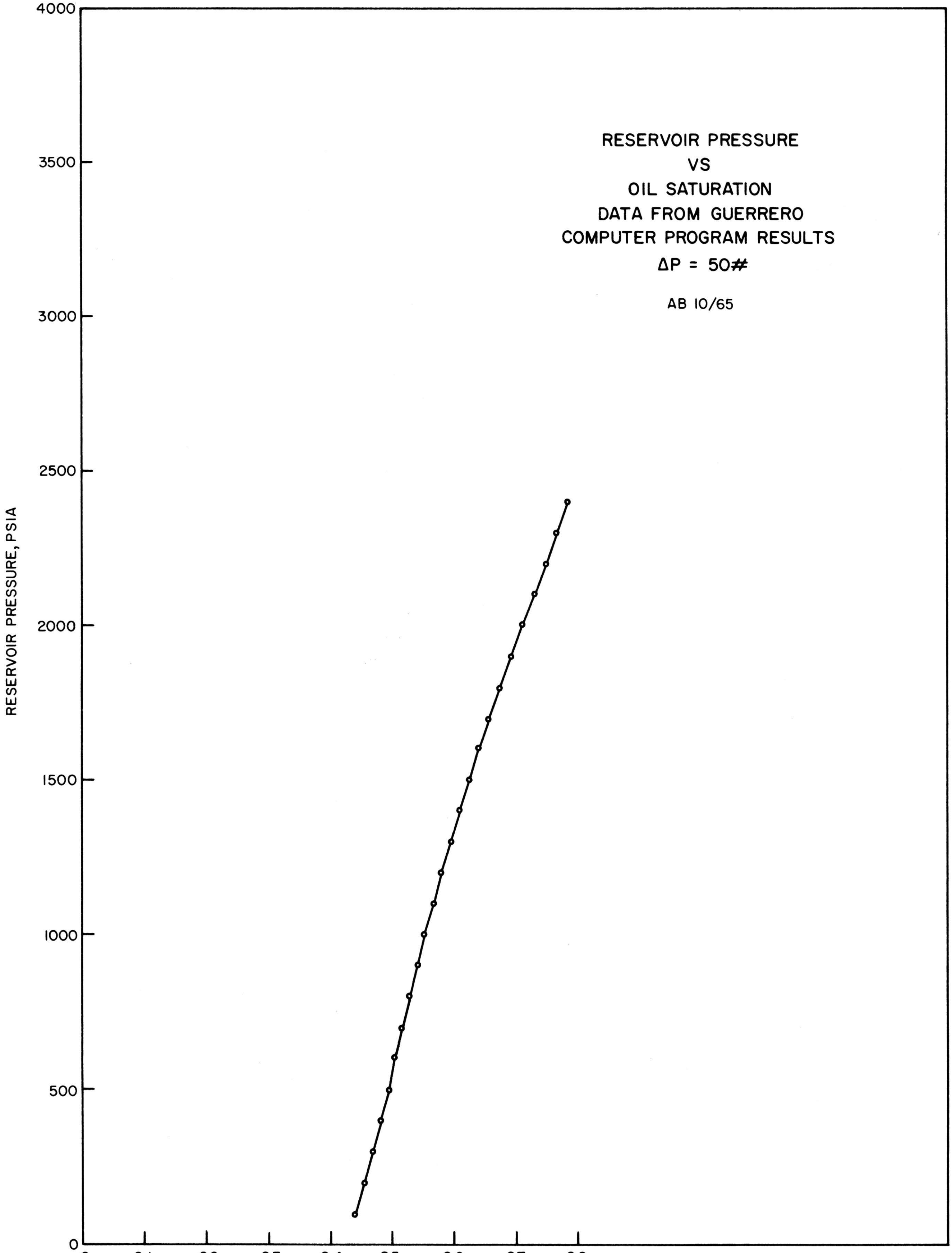
PRES PSIA	OIL FVF BBL/STB	GAS FVF SCF/BBL	SOL GOR SCF/STB	PRO GOR SCF/STB	OIL SATUR
2500	1.498	954	721	721	0.8000
2300	1.463	866	669	669	0.7682
2100	1.429	781	617	684	0.7302
1900	1.395	694	565	1342	0.6925
1700	1.361	612	513	1902	0.6586
1500	1.327	531	461	2399	0.6266
1300	1.292	453	409	3050	0.5964
1100	1.258	377	357	3709	0.5686
900	1.224	303	305	4376	0.5425
700	1.190	232	253	4789	0.5177
500	1.156	162	201	4822	0.4934
300	1.121	96	149	4215	0.4684
100	1.087	31	97	2144	0.4399

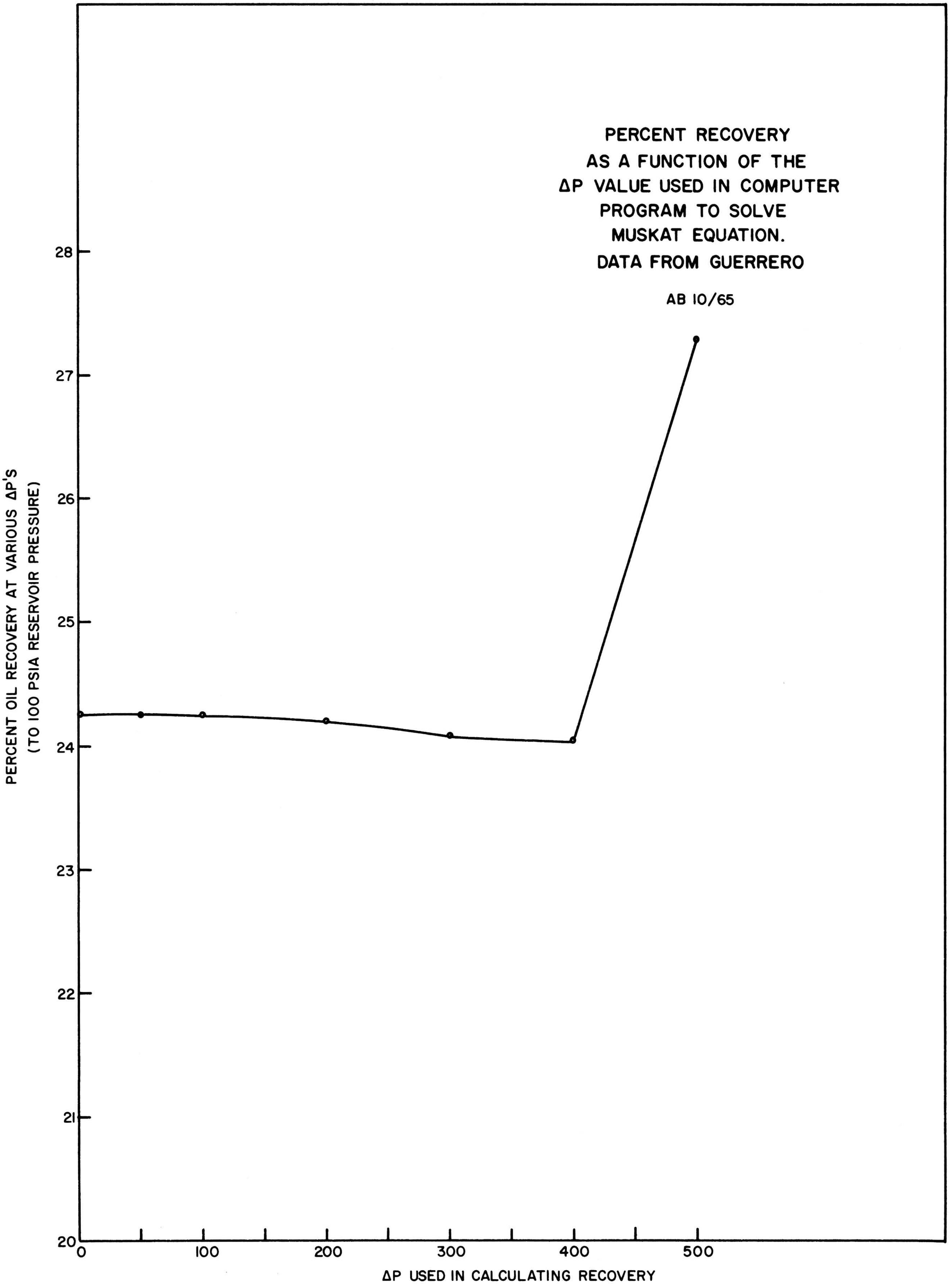
KG/KO	VISCOS OIL CP	VISCOS GAS CP	CUM OIL BBLS	CUM GAS MCF
0.0000	0.49	0.0170	0.0000000@+00	0.0000000@+00
0.0000	0.54	0.0166	1.6797803@-02	1.1645619@-02
0.0016	0.60	0.0162	4.3166786@-02	2.8568659@-02
0.0192	0.66	0.0158	7.0450051@-02	5.6117471@-02
0.0352	0.73	0.0154	9.3888097@-02	9.4118193@-02
0.0516	0.80	0.0150	1.1576982@-01	1.4099319@-01
0.0740	0.89	0.0146	1.3561195@-01	1.9477687@-01
0.1024	0.98	0.0142	1.5362165@-01	2.5550540@-01
0.1390	1.09	0.0138	1.6999829@-01	3.2177987@-01
0.1835	1.20	0.0134	1.8530938@-01	3.9226706@-01
0.2430	1.32	0.0130	2.0080121@-01	4.6735260@-01
0.3260	1.46	0.0126	2.1752717@-01	5.4481014@-01
0.4574	1.62	0.0122	2.4214299@-01	6.2658738@-01

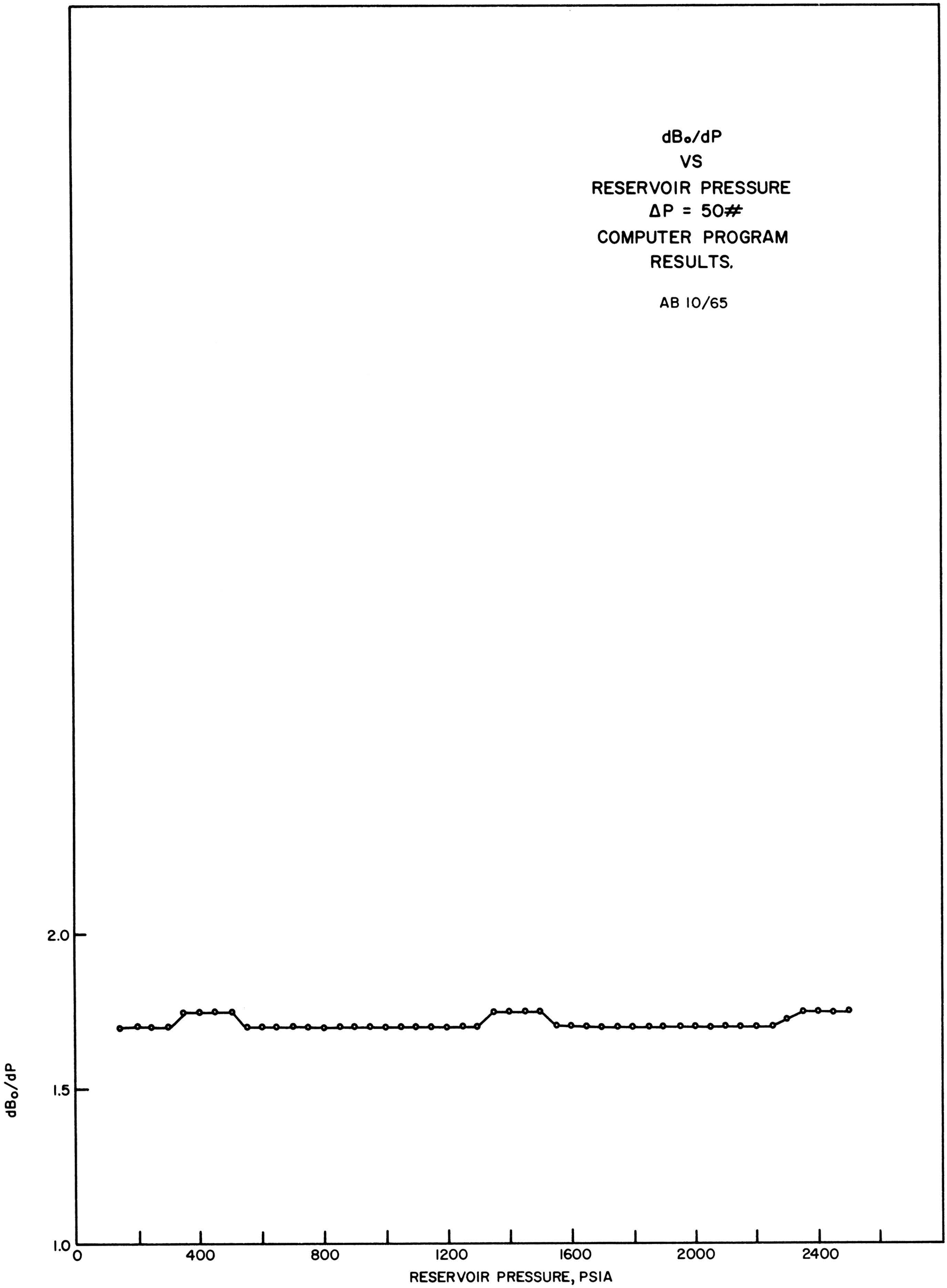

RESERVOIR PRESSURE
AND
PRODUCING GAS-OIL RATIO
VS
PERCENT RECOVERY
DATA FROM GUERRERO
COMPUTER PROGRAM RESULTS
ΔP = 50#
AB 10/65
PRESSURE
GOR
RESERVOIR PRESSURE, PSIA
PRODUCING GOR SCF/STB
CUMULATIVE OIL RECOVERY %
0
500
1000
1500
2000
2500
3000
3500
4000
1000
2000
3000
4000
5000
6000
7000
8000
2
4
6
8
10
12
14
16
18
20
22
24
26
28



RESERVOIR PRESSURE, PSIA
VS
PRODUCED GAS, MCF
FOR RESERVOIR CONTAINING
1 STB OIL DATA FROM
GUERRERO. ΔP = 50#
COMPUTER PROGRAM RESULTS
AB 10/65
RESERVOIR PRESSURE, PSIA
4000
3500
3000
2500
2000
1500
1000
500
0
0
100
200
300
400
500
600
700
CUMULATIVE GAS PRODUCED, MCF


RESERVOIR PRESSURE
VS
OIL SATURATION
DATA FROM GUERRERO
COMPUTER PROGRAM RESULTS
ΔP = 50#
AB 10/65
RESERVOIR PRESSURE, PSIA
4000
3500
3000
2500
2000
1500
1000
500
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
OIL SATURATION


PERCENT RECOVERY
AS A FUNCTION OF THE
ΔP VALUE USED IN COMPUTER
PROGRAM TO SOLVE
MUSKAT EQUATION.
DATA FROM GUERRERO
AB 10/65
PERCENT OIL RECOVERY AT VARIOUS ΔP'S
(TO 100 PSIA RESERVOIR PRESSURE)
28
27
26
25
24
23
22
21
20
0
100
200
300
400
500
ΔP USED IN CALCULATING RECOVERY



dBo/dP
VS
RESERVOIR PRESSURE
ΔP = 50#
COMPUTER PROGRAM
RESULTS.
AB 10/65
2.0
1.5
1.0
dBo/dP
0
400
800
1200
1600
2000
2400
RESERVOIR PRESSURE, PSIA

APPENDIX F.- Theoretical derivation of Muskat's Equation (Craft and Hawkins, 1959).

In order to derive the Muskat Equation for prediction of a solution-gas drive reservoir the following assumptions are needed:

(a) Uniformity of reservoirs at all times regarding porosity, fluid saturations, and relative permeability. Studies have shown that gas and oil saturations about wells are surprisingly uniform at all stages of depletion.
(b) Uniform pressure throughout the reservoir in both gas and oil zones. This means that the gas and oil volume factors, the gas and oil viscosities, and the solution gas are the same throughout the reservoir.
(c) Negligible gravity segregation forces.
(d) Equilibrium at all times between the gas and oil phases.
(e) A gas liberation mechanism which is the same as that used to determine the fluid properties.
(f) No water encroachment and negligible water production.
(g) The reservoir is not above its bubble point pressure.

Assume a reservoir with a pore volume of V_p, barrels, and an oil saturation S_o, and an oil formation volume factor of B_o, barrels per stock tank barrel. Then the remaining oil at any pressure P in psia will be:

$$N_r = \frac{S_o V_p}{B_o} \quad \text{stock tank barrels.} \tag{1}$$

If this equation is differentiated with respect to pressure to get incremental oil production, the result is:

$$\frac{dN_r}{dP} = V_p \left(\frac{1}{B_o} \frac{dS_o}{dP} - \frac{S_o}{B_o^2} \frac{dB_o}{dP} \right) \tag{2}$$

The remaining standard cubic feet of gas at pressure P, both dissolved and free, is:

$$G_r = \frac{R_s V_p S_o}{B_o} + (1 - S_o - S_w) B_g V_p \tag{3}$$

where R_s is the solubility of the gas in oil, SCF/STB, at pressure P, and B_g is the gas formation volume factor, SCF/BBL, at pressure P, and S_w is the constant connate water saturation. The rate of change of G_r with respect to pressure is:

$$\frac{dG_r}{dP} = V_p \left(\frac{R_s}{B_o} \frac{dS_o}{dP} + \frac{S_o}{B_o} \frac{dR_s}{dP} - \frac{R_s S_o}{B_o^2} \frac{dB_o}{dP} + (1 - S_o - S_w) \frac{dB_g}{dP} - B_g \frac{dS_o}{dP} \right) \tag{4}$$

If the reservoir pressure is dropping at the rate dP/dt, the current or producing gas-oil ratio, R, at this pressure, is:

$$R = \frac{\Delta G_p}{\Delta N_p} = \frac{\Delta G_r}{\Delta N_r} = \frac{\Delta G_r / \Delta P}{\Delta N_r / \Delta P} = \frac{dG_r/dP}{dN_r/dP} \tag{5}$$

Substituting equations (2) and (4) in equation (5)

$$R = \frac{\dfrac{R_s dS_o}{B_o dP} + \dfrac{S_o dR_s}{B_o dP} - \dfrac{R_s S_o dB_o}{B_o^2 dP} + (1-S_o-S_w) \dfrac{dB_g}{dP} - B_g \dfrac{dS_o}{dP}}{\dfrac{1}{B_o} \dfrac{dS_o}{dP} - \dfrac{S_o}{B_o^2} \dfrac{dB_o}{dP}} \tag{6}$$

Equation (6) is an expression of the producing gas-oil ratio, derived from the material balance equation which is given below:

$$N_o = \frac{N_p \; [B_o + B_g \; (R_p - R_s)]}{B_o - B_{oi} + B_g \; (R_{si} - R_s)} \tag{7}$$

where N_o is the initial oil in place in stock tank barrels, R_p is the cumulative produced gas-oil ratio in SCF/STB and the i subscripts indicate initial conditions.

The producing gas-oil ratio, R, may also be derived from the steady-state flow equations for oil and gas. If q_g is the flow of gas expressed at mean reservoir pressure and reservoir temperature

$$q_g = \frac{7.08\, k_g h(P_e - P_w)}{\mu_g \ln (r_e/r_w)} \tag{8}$$

where k_g is the permeability to gas, P_e is the external reservoir boundary pressure, psia, h, net pay thickness, feet, P_w is the bottom hole pressure, psia, μ_g is the viscosity of the gas, centipoises, r_e is the external reservoir boundary radius, in feet, and r_w is the well bore radius, in feet. Similarly the flow rate of oil q_o expressed in reservoir barrels per day is:

$$q_o = \frac{7.08 k_o h(P_e - P_w)}{\mu_o \ln (r_e/r_w)} \tag{9}$$

where μ_o is the oil viscosity in centipoises and k_o is the permeability to oil. The producing gas-oil ratio can be expressed in surface units of SCF/STB as:

$$\left(\frac{q_g}{q_o}\right) sc = \frac{q_g B_g}{q_o/B_o} = B_o B_g \frac{k_g}{k_o} \frac{\mu_o}{\mu_g} \tag{10}$$

where B_g is in SCF/BBL. Because equation (8) applies only to the flowing free gas and not to the solution gas which flows to the well bore with the oil, equation (10) must be increased by the solution gas-oil ratio to give the total surface producing gas-oil ratio, R, in SCF/STB.

$$R = B_o B_g \frac{k_g}{k_o} \frac{\mu_o}{\mu_g} + R_s \tag{11}$$

Equation (11) may now be equated to equation (6) and solved for d S_o/dP to give:

$$\frac{dS_o}{dP} = \frac{\frac{S_o}{B_o B_g} \frac{dR_s}{dP} + \frac{S_o}{B_g} \frac{k_g}{k_o} \frac{\mu_o}{\mu_g} \frac{dB_o}{dP} + (1 - S_o - S_w) \frac{1}{B_g} \frac{dB_g}{dP}}{1 + \frac{k_g}{k_o} \frac{\mu_o}{\mu_g}} \tag{12}$$

Equation (12) is known as the Muskat Equation for a solution gas drive reservoir. Hoss (1948) extended this equation to include a gas cap term and the effect of gas injection operations. To make this extension, the following additional assumptions are required:

(a) the gas cap does not expand and,
(b) the injected gas is distributed uniformly throughout the producing horizon.

If r is the fraction of produced gas which is injected, R, the total producing gas-oil ratio at any pressure P, and S_{oi} the initial oil saturation in the gas cap, and m is the ratio of initial gas cap volume to initial oil volume, the equation takes on the following form:

$$\frac{dS_o}{dP} = \left(\frac{1}{1 + \frac{k_g}{k_o}\frac{\mu_o}{\mu_g} - \frac{rR}{B_oB_g}}\right)\left\{\frac{S_o}{B_oB_g}\frac{dR_s}{dP} + S_o\left(\frac{k_g}{k_o} - \frac{rR}{B_oB_g}\frac{\mu_g}{\mu_o}\right)\frac{1}{B_o}\frac{\mu_o}{\mu_g}\frac{dB_o}{dP} + \right.$$

$$\left.\left[m\left(1-S_w - \frac{S_{oi}B_o}{B_{oi}}\right) + (1-S_w-S_o)\right]\frac{1}{B_g}\frac{dB_g}{dP} - m\frac{S_{oi}}{B_{oi}}\frac{dB_o}{dP}\right\} \tag{13}$$

If there is no gas cap or no gas injection, then m = 0 and r = 0 and this equation will reduce to equation (12).

The Muskat Equation is most frequently used to predict the oil and gas production and producing gas-oil ratio as a function of pressure during the pressure decline of a reservoir. The cumulative oil production, N_p, is calculated from the saturation equation:

$$N_p = N_o - N_o\left(\frac{S_o\,B_{oi}}{S_{oi}B_o}\right) \tag{14}$$

To obtain G_p, the cumulative gas produced in SCF, the two-phase formation volume factor, B_t, in STB/BBL must first be computed as:

$$B_t = B_o + (R_{si} - R_s)/B_g \tag{15}$$

then,

$$G_p = B_g\left[N_o\left((B_t - B_{oi}) + \frac{m\,B_{oi}\,(B_{gi} - B_g)}{B_g}\right) - N_p\left(B_t - \frac{R_{si}}{B_g}\right)\right] \tag{16}$$

R, the producing gas-oil ratio is computed using equation (11).

The solution of the Muskat Equation for this program gives S_o, N_p, G_p, and R at a series of pressures beginning with the initial pressure and decreasing to P_{min} at intervals of ΔP.

APPENDIX G.- Procedure LAGDUAL

This procedure utilizes Lagrange polynomial interpolation to calculate the value of a dependent variable y for a given independent variable x, where x lies within the range of x_i $i = 1, 2, 3, \ldots n$.

This procedure also allows the option of computing an approximation to the first derivative of the function, dy/dx.

The general form of the Lagrange equation for n'th order interpolation is as follows:

Let $(x_o, y_o), (x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)$ denote n + 1 corresponding pairs of values of any two variables x and y where y = f(x). If the value of y is desired at any value of x, the Lagrange equation for determining this value with an n order polynomial is as follows:

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)\ldots\ldots(x_0-x_n)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\ldots\ldots(x_1-x_n)} y_1 +$$

$$\frac{(x-x_0)(x-x_1)(x-x_3)\ldots\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)\ldots\ldots(x_2-x_n)} y_2 + \ldots\ldots \frac{(x-x_0)(x-x_1)(x-x_2)\ldots\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)(x_n-x_2)\ldots\ldots(x_n-x_{n-1})} y_n \qquad (1)$$

The more condensed form of the above general expression

$$y = \sum_{i=1}^{n} \frac{\prod_{i=1}^{n} (x-x_i)}{\prod_{i=1}^{n} (x_j-x_i)} y_i \qquad \Pi = \text{repeated product symbol} \quad j \neq i \qquad (2)$$

To compute the first derivative of the polynomial, equation (2) takes the form:

$$y^1 = \sum_{j=1}^{n} \frac{\sum_{i=1}^{n} \prod_{k=1}^{n} (x-x_k)}{\prod_{i=1}^{n} (x_j-x_i)} y_j \qquad i \neq j \quad k \neq j \qquad (3)$$

The expanded form for a 3rd order is:

$$y^1 = \frac{(x-x_3)(x-x_4) + (x-x_2)(x-x_3) + (x-x_2)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} y_1 + \frac{(x-x_1)(x-x_3) + (x-x_1)(x-x_4) + (x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} y_2 +$$

$$\frac{(x-x_1)(x-x_2) + (x-x_1)(x-x_4) + (x-x_2)(x-x_4)}{(x_3-x_1)(x_3-x_2)(x_3-x_4)} y_3 + \frac{(x-x_1)(x-x_2) + (x-x_1)(x-x_3) + (x-x_2)(x-x_3)}{(x_4-x_1)(x_4-x_2)(x_4-x_3)} y_4 . \qquad (4)$$

The procedure is so written that the polynomial order for interpolation can be specified by the user.

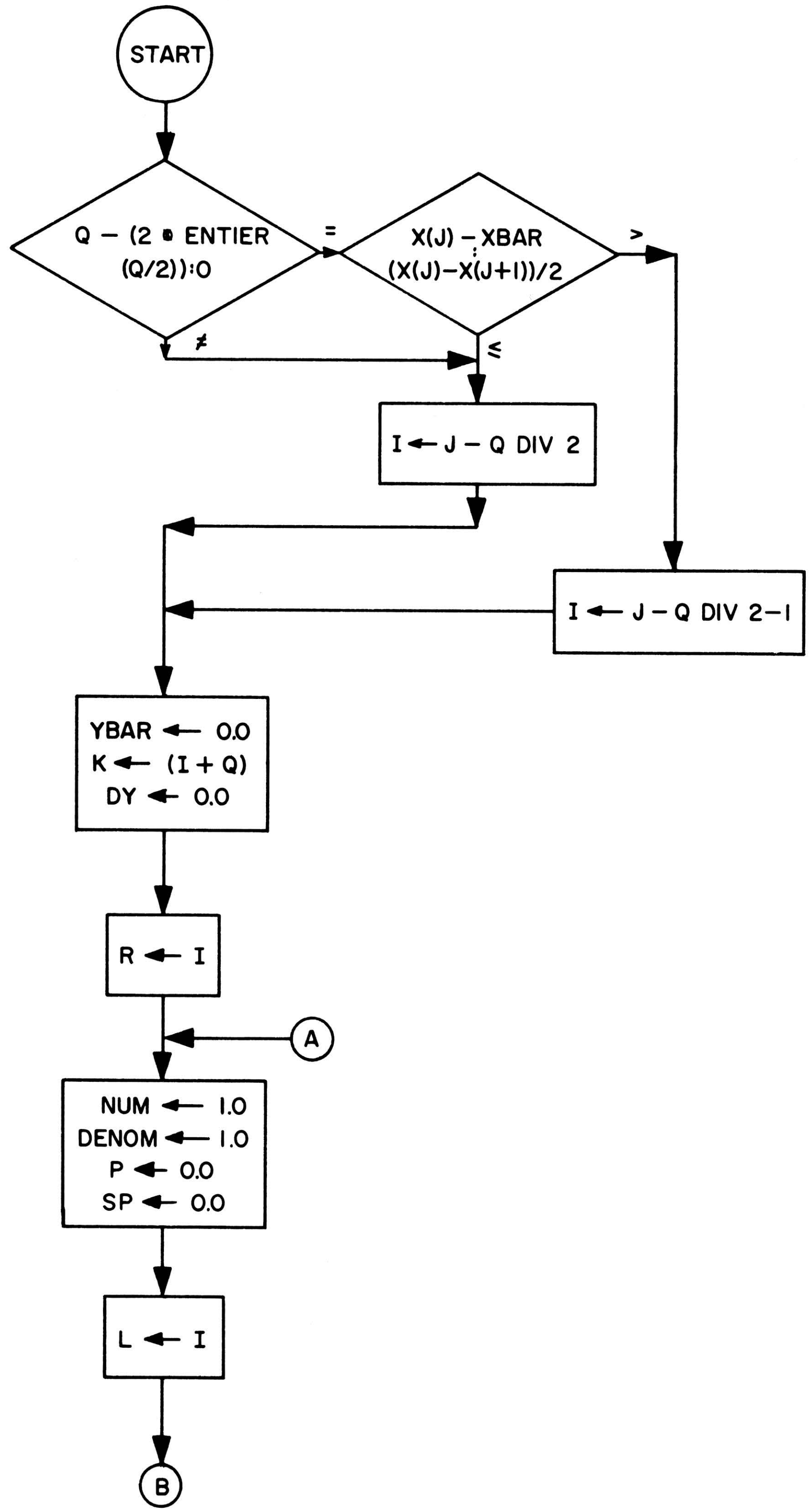
The program does not extrapolate the table of values beyond its given range. However, if the value of x lies so close to one of the table extremities that insufficient points are available for computing y at the polynomial order, n is successively reduced until an order is found which will permit calculation of y with the available data.

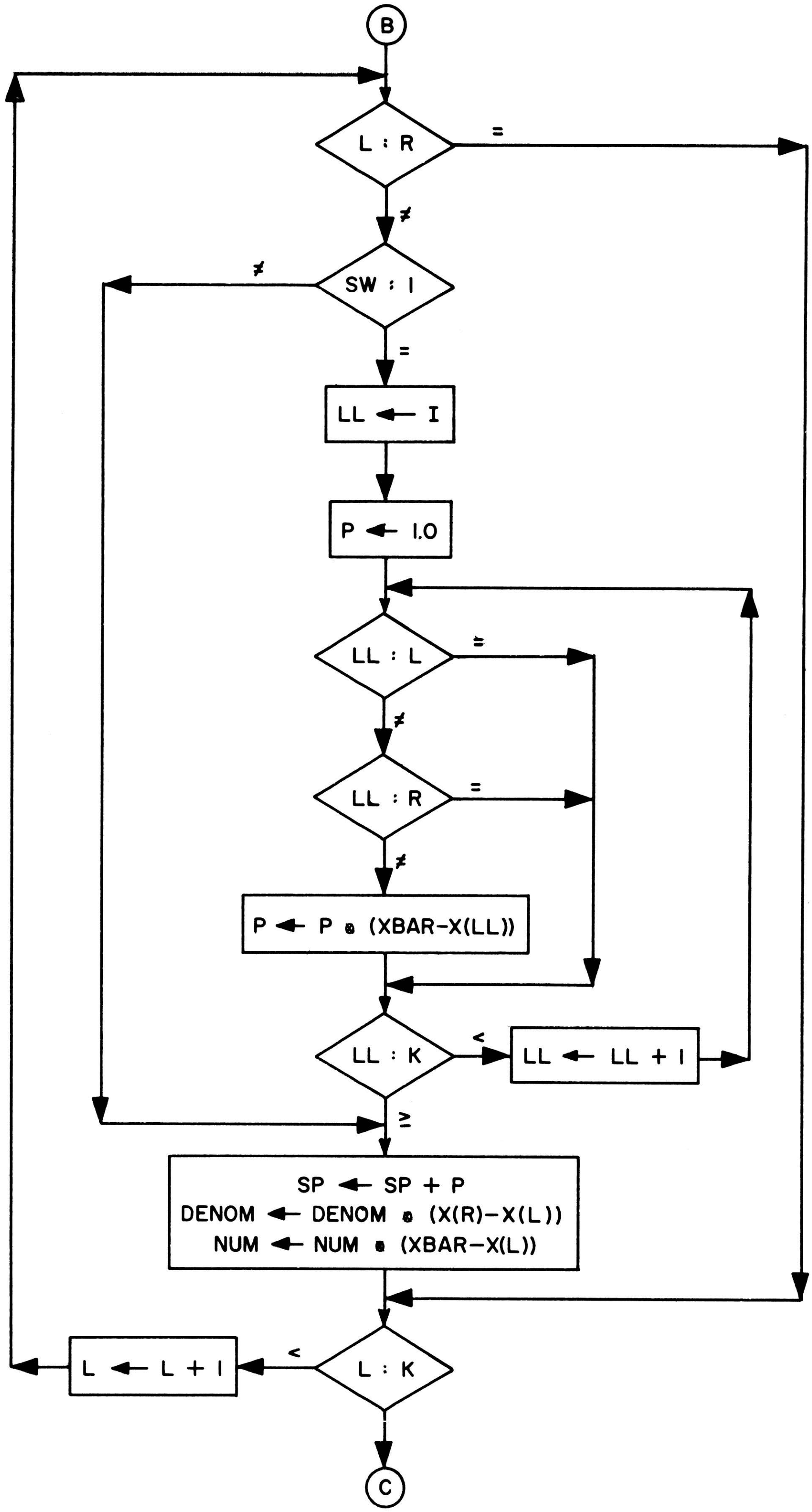
The procedure name and argument is given below. The flow chart and table of nomenclature follow:

Procedure LAGDUAL (Q, J, X, Y, XBAR, YBAR, DY, SW, NUMB) where,

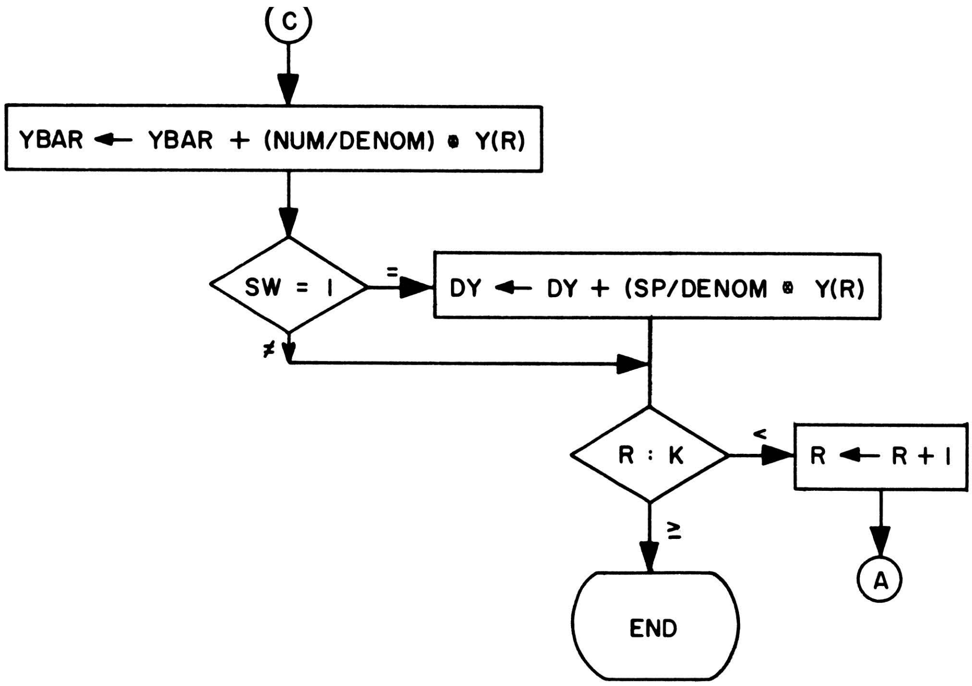
Q	order of interpolation.
J	subscript of table value immediately above x.
X	array of independent variables.
Y	array of dependent variables.
XBAR	the value at which interpolation is required.
YBAR	the required interpolation.
DY	the first derivative of y with respect to x.
SW	an integer used to indicate whether DY is desired or not. If SW = 1 then DY is computed in addition to YBAR.
NUMB	number of values in table.

FLOW CHART FOR PROCEDURE
LAGDUAL (Q, J, X, Y, XBAR, YBAR, DY, SW)





B
L : R
=
≠
SW : I
≠
=
LL ← I
P ← 1.0
LL : L
=
≠
LL : R
=
≠
P ← P * (XBAR-X(LL))
LL : K
<
LL ← LL + I
≥
SP ← SP + P
DENOM ← DENOM * (X(R)-X(L))
NUM ← NUM * (XBAR-X(L))
L : K
<
L ← L + I
C


C
YBAR ← YBAR + (NUM/DENOM) * Y(R)
SW = 1
=
DY ← DY + (SP/DENOM * Y(R)
≠
R : K
<
R ← R + 1
≥
A
END

PROCEDURE LAGDUAL (Q, J, X, Y, XBAR, YBAR, DY, SW, NUMB):

TABLE OF NOMENCLATURE

Algebraic symbol	ALGOL symbol	Type	Description and units
dy/dx	DY	REAL	The first derivative of the dependent variable with respect to the independent variable.
i	I	INTEGER	Subscript of the first value of x in the interpolation interval for Q order interpolation.
j	R	INTEGER	A counter which is the subscript of the y's in the interpolation and first derivative Lagrange equations.
j	J	INTEGER	Integer used to locate the interpolating interval.
L	L	INTEGER	A counter.
LL	LL	INTEGER	A counter.
n	K	INTEGER	Subscript of the last value of x in the interpolation interval for Q order interpolation.
	NUMB	INTEGER	Number of table values.
P	P	REAL	The individual products of the differences in the Lagrange first derivative equation.
Q	Q	INTEGER	Order of interpolation desired.
SP	SP	REAL	The sums of the products in the numerator of the Lagrange equation for computing the first derivative.
SW	SW	INTEGER	If a "1" is put in for SW then YBAR and DY are the output. If anything else is used only YBAR will be the output from LAGDUAL.
X	X	REAL	Array of independent variables.
$\bar{X}$	XBAR	REAL	Value at which interpolation is desired.
Y	Y	REAL	Array of dependent variables.
$\bar{Y}$	YBAR	REAL	The desired interpolation.
	DENOM	REAL	The denominator in the Lagrange equations for polynomial interpolation and differentiation.
	NUM	REAL	The sum of the numerator terms in the Lagrange interpolation equation.

APPENDIX H.- Runge-Kutta Method.

Equation (12) is an ordinary differential equation of the first order (Appendix F). To solve this equation with digital computers, it is necessary to utilize a numerical method. In this ALGOL program, a third order RUNGE-KUTTA predictor-corrector technique is employed. This technique is outlined in Hildebrand (1956) and a brief discussion of this method is presented here.

Let $dy/dx = f(x,y)$ represent any first order differential equation and let h denote the interval between successive values of x. Then if an initial point, x_0, y_0 is known, the next value of y, i.e. y, corresponding to $x_0 + h$ may be computed from the following:

$$k_1 = f(x_0, y_0)h \tag{1}$$

$$k_2 = f(x_0 + h/3, y_0 + k_1/3)h \tag{2}$$

$$k_3 = f(x_0 + 2/3h, y_0 + 2/3k_2)h \tag{3}$$

$$\Delta y = 1/4(k_1 + 3k_3) \tag{4}$$

$$y_1 = y_0 + \Delta y, \quad x_1 = x_0 + h \tag{5}$$

The general form of the series of equations is

$$k_i = f(x_0 + (i-1/3)h, \quad y_0 + (i-1/3)k_{i-1})\, h \tag{6}$$

The following is a step by step process of this method.

Step 1. Evaluate the slope ($k_1/\Delta x$) at the point (x_0, y_0) $\quad k_1/\Delta x = f(x_0, y_0)$

Step 2. Using the slope from Step 1, and starting at (x_0, y_0) evaluate the slope ($k_2/\Delta x$) at ($x_0 - \Delta x/3$, $y_0 - k_1/3$). This point is the first approximation of the change in y. $k_2/\Delta x = f(x_0 - \Delta x/3, y_0 - k_1/3)$.

Step 3. Using the slope from Step 2, and again starting from (x_0, y_0) evaluate the slope ($k_3/\Delta x$) at ($k_0 - 2/3\Delta x$, $y_0 - 2/3k_2$). This is the second approximation of the change in y. $k_3/\Delta x = f(x_0 - 2/3\Delta x, y_0 - 2k_2/3)$.

Step 4. The slopes are then weighted by the equation $1/4(k_1 + 3k_3)$ and the weighted average is used as the change in y (There are several weighting techniques, but as pointed out by Hildebrand, the accuracy is about the same.)

Step 5. Using y at $y_0 - \Delta y$ the process is repeated with the new x_0, y_0 now becoming our point determined above ($x_0 = x_0 - \Delta x$ and $y_0 = y_0 - \Delta y$).

KANSAS GEOLOGICAL SURVEY COMPUTER PROGRAM
THE UNIVERSITY OF KANSAS, LAWRENCE

PROGRAM ABSTRACT

Title (If subroutine state in title):

Prediction of the performance of a solution gas drive reservoir by Muskat's Equation

Computer: Burroughs B5500 Date: January 4, 1967

Programming language: ALGOL

Author, organization: Apolonio Baca - Reserves Engineer
Northern Natural Gas Company

Direct inquiries to: Author, or

Name: Daniel F. Merriam Address: Kansas Geological Survey, Univ. of Kansas
Lawrence, Kansas 66044

Purpose/description: To predict the performance of a solution gas drive reservoir using the Muskat Equation. The reservoir does not have a gas cap and does not have water encroachment. The term B_g may be known or can be internally computed using the optional input.

Mathematical method: The Muskat Equation is solved through the use of the Runge-Kutta numerical technique for solving differential equations. The Lagrange polynomial interpolation method is used to perform interpolations and to obtain the necessary first order differentials.

Restrictions, range: See description of program section for details on limitations and restrictions.

Storage requirements: Estimated Core Storage 6,724 words

Equipment specifications: Memory 20K ___ 40K ___ 60K ___ K ___

Automatic divide: Yes ___ No ___ Indirect addressing Yes ___ No ___

Other special features required ___

Additional remarks (include at author's discretion: fixed/float, relocatability; optional: running time, approximate number of times run successfully, programming hours) ___

COMPUTER CONTRIBUTIONS

Kansas Geological Survey
University of Kansas
Lawrence, Kansas

Computer Contribution

1. Mathematical simulation of marine sedimentation with IBM 7090/7094 computers, by J.W. Harbaugh, 1966. . . . . $1.00
2. A generalized two-dimensional regression procedure, by J.R. Dempsey, 1966 . . . . . . $0.50
3. FORTRAN IV and MAP program for computation and plotting of trend surfaces for degrees 1 through 6, by Mont O'Leary, R.H. Lippert, and O.T. Spitz, 1966 . . . . $0.75
4. FORTRAN II program for multivariate discriminant analysis using an IBM 1620 computer, by J.C. Davis and R.J. Sampson, 1966 . . . . $0.50
5. FORTRAN IV program using double Fourier series for surface fitting of irregularly spaced data, by W.R. James, 1966 . . . . $0.75
6. FORTRAN IV program for estimation of cladistic relationships using the IBM 7040, by R.L. Bartcher, 1966 . . . . $1.00
7. Computer applications in the earth sciences: Colloquium on classification procedures, edited by D.F. Merriam, 1966 . . . . $1.00
8. Prediction of the performance of a solution gas drive reservoir by Muskat's Equation, by Apolonio Baca, 1967 . . . . $1.00

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Fourier series characterization of cyclic sediments for stratigraphic correlation, by F.W. Preston and J.H. Henderson (reprinted from Symposium on cyclic sedimentation, D.F. Merriam, editor, Kansas Geological Survey Bulletin 169, v. 2, 1964)

Geology and the computer, by D.F. Merriam (reprinted from New Scientist, v. 26, no. 444, 1965)

Quantitative comparison of contour maps, by D.F. Merriam and P.H.A. Sneath (reprinted from Journal of Geophysical Research, v. 71, no. 4, 1966)

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Geologic use of the computer, by D.F. Merriam (reprinted from Wyoming Geol. Assoc., 20th Field Conf., 1966)