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# Exclusive $\eta_c$ production by $\gamma^*\gamma$ interactions in electron-ion collisions

I. Babiarz[a], V. P. Goncalves[b], W. Schäfer[a], A. Szczurek [a,c]
[a] *Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, PL-31-342 Kraków, Poland*
[b] *Physics and Mathematics Institute, Federal University of Pelotas, Postal Code 354, 96010-900, Pelotas, RS, Brazil*
[c] *College of Mathematics and Natural Sciences, University of Rzeszów, ul. Pigonia 1, PL-35-310 Rzeszów, Poland*

One of the main goals of future electron-ion colliders is to improve our understanding of the structure of hadrons. We study the exclusive $\eta_c$ production by $\gamma\gamma^*$ interactions in eA collisions and demonstrate that future experimental analysis of this process can be used to improve the description of the transition form factor. The rapidity, transverse momentum and photon virtuality distributions are estimated considering the energy and target configurations expected to be present at the EIC, EicC and LHeC and assuming different predictions for the light-front wave function of the meson. Our results indicate that the electron-ion colliders can be considered an alternative for providing supplementary data to those obtained in colliders.

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## 1 Introduction

Studies of transition form factors in various processes can provide complementary information on quarkonium structure and exotic state compositions. We shortly remind the clue of the light front wave function approach (LFWF) to the photon-photon transition form factor. In Fig. 1, we depict an example of photon-photon fusion in $e^+e^-$ collision. We consider the space-like region, where both photons are off-shell. In the LFWF approach, we assume that the dominant contribution in the light front Fock state expansion is the $\mathsf{Q}\bar{\mathsf{Q}}$ component:

$$|\eta_c; P_+, \vec{P}_\perp\rangle = \sum_{i,j,\lambda,\bar\lambda} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2\vec{k}_\perp}{z(1-z)16\pi^3} \Psi_{\lambda\bar\lambda}(z,\vec{k}_\perp) \big|\mathsf{Q}_{i\lambda}(zP_+, \vec{p}_{\perp c}) \bar{\mathsf{Q}}^j_{\bar\lambda}((1-z)P_+, \vec{p}_{\perp\bar{c}})\big\rangle + \dots \quad (1)$$

Thus we consider the $\mathsf{Q}\bar{\mathsf{Q}}$ bound system in colour-singlet configuration. Here $\Psi_{\lambda\bar\lambda}(z,\vec{k}_\perp)$ is the corresponding light front wave function. In ref.[1,2], we describe in detail the procedure to construct the spin-orbit part of the rest frame $c\bar{c}$ wave function and the transformation to light-front via Melosh spin-rotation matrix. The radial part is the solution of the Schrödinger equation for five potential models from the literature, mapped to light-front representation. In the current analysis, we also employ a wave function obtained from the Basis Light Front Quantisation (BLFQ) approach [3,4,5].

We can define transition form factor via covariant amplitude $\mathcal{M}_{\mu\nu}$ of the photon-photon to quark-antiquark:

$$\mathcal{M}_{\mu\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c) = 4\pi\alpha_{\rm em}\,(-i)\varepsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta\, F(Q_1^2, Q_2^2)\,. \quad (2)$$

Above we denote photons four momenta as $q_1, q_2$, and $\varepsilon_{\mu\nu\alpha\beta}$ is the antisymmetric tensor. Remaining indices $\mu\nu$ refer to photon's helicties $e_1^\mu(\lambda_1)$, $e_2^\nu(\lambda_2)$, which depend on the chosen frame. In Refs. [1,6], we

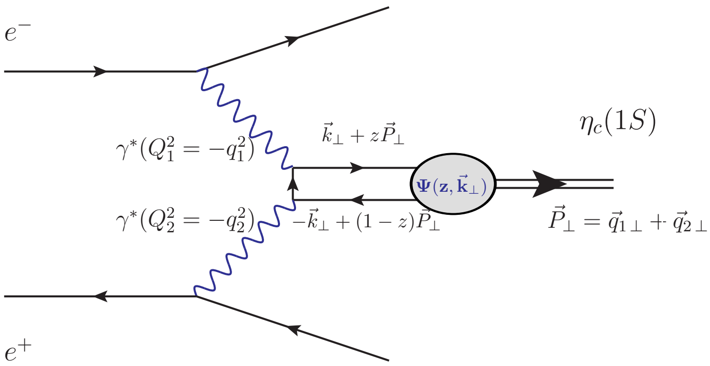


Figure 1 – The sketch of the photon-photon interaction in $e^+e^-$ collision. Here photons are space-like, their virtualities: $Q^2_{1;2} > 0$. Quark Q, and antiquark $\bar{\mathsf{Q}}$ carry the fraction of the meson $P_+$ momenta $z$ and $(1-z)$, respectively, $\vec{k}_\perp$ is relative momentum between quark and antiquark.

put into evidence that in the limit for $Q_2^2 \to 0$, transition form factor takes the form:

$$F(Q^2, 0) = e_c^2 \sqrt{N_c}\, 4 \int \frac{dz d^2\vec{k}_\perp}{\sqrt{z(1-z)} 16\pi^3} \left\{ \frac{\tilde{\psi}_{\uparrow\downarrow}(z, k_\perp)}{\vec{k}_\perp^{\ 2} + \mu^2} + \frac{\vec{k}_\perp^{\ 2}}{[\vec{k}_\perp^{\ 2} + \mu^2]^2} \Big( \tilde{\psi}_{\uparrow\downarrow}(z, k_\perp) + \frac{m_c}{k_\perp} \tilde{\psi}_{\uparrow\uparrow}(z, k_\perp) \Big) \right\}, \quad (3)$$

with $\mu^2 = z(1-z)Q^2 + m_c^2$, and the helicity components $\tilde{\psi}_{\uparrow\downarrow}(z, k_\perp), \tilde{\psi}_{\uparrow\uparrow}(z, k_\perp)$ are related to the same radial wave function $\psi(z, k_\perp)$ as:

$$\tilde{\psi}_{\uparrow\downarrow}(z, k_\perp) \to \frac{m_c}{\sqrt{z(1-z)}} \psi(z, k_\perp), \quad \text{and} \quad \tilde{\psi}_{\uparrow\uparrow}(z, k_\perp) \to \frac{-|\vec{k}_\perp|}{\sqrt{z(1-z)}} \psi(z, k_\perp), \quad (4)$$

so that there appears cancellation of the terms in the round brackets in Eq. (3) [1]. Other approaches, such as the BLFQ approach of [3,4] do not guarantee this cancellation. In particular, at the on-shell point, where two photons are real, the transition form factor is related to radiative decay width:

$$\Gamma_{\gamma\gamma\to\eta_c} = \frac{\pi}{4} \alpha_{em}^2 M_{\eta_c}^3 |F(0,0)|^2, \quad (5)$$

with $\alpha_{em}$ being fine-structure constant.

## 2 Cross-section for one virtual photon in eA collision

The contribution of $\eta_c$ production associated with photon-photon interactions is similar to a proton target. It dominates when an ion is present, which is directly associated with nuclear charge squared ($Z^2$) enhancement present in the nuclear photon flux. This point is the main motivation for exclusive analysis of $\eta_c$ production, which can occur via photon-photon fusion in an electron-ion collision.

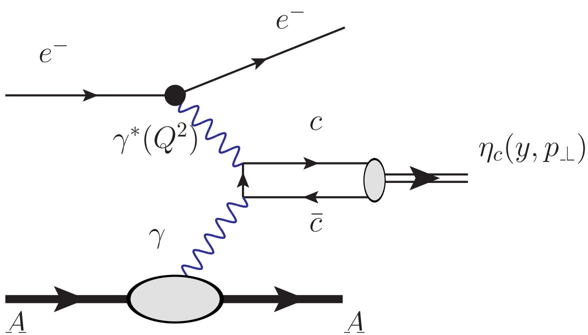


Figure 2 – Example illustration of an electron-ion collision involving one virtual photon. In particular we consider ions: $^{197}$Au or $^{208}$Pb.

Here we are interested in the process, where one of the photons has virtuality $Q^2$, and the second emitted by the ion is quasi-real, see Fig. 2. In order to describe exclusive production we can take advantage of the factorisation formula

$$\sigma(eA \to e\eta_c A) = \int d\omega_e dQ^2 \frac{d^2 N_e}{d\omega_e dQ^2} \sigma(\gamma^* A \to \eta_c A)\,, \tag{6}$$

where the electron flux factor is given by [7]

$$\frac{d^2 N_e}{d\omega_e dQ^2} = \frac{\alpha_{em}}{\pi \omega_e Q^2}\left[\left(1-\frac{\omega_e}{E_e}\right)\left(1-\frac{Q^2_{min}}{Q^2}\right)+\frac{\omega_e^2}{2E_e^2}\right]. \tag{7}$$

Here we denote $\omega_e$ as the energy of the photon emitted by the electron with energy $E_e$. Constrains by the maximum of the electron energy loss imply that $Q^2_{min} = m_e^2\omega_e^2/[E_e(E_e-\omega_e)]$ and $Q^2_{max} = 4E_e(E_e-\omega_e)$. If we assume that the production of $\eta_c$ is dominated by the subprocess $\gamma^*\gamma \to \eta_c$, we can apply the equivalent photon approximation in the form:

$$\sigma(\gamma^* A \to \eta_c A) = \int d\omega_A \frac{dN}{d\omega_A}\sigma_{\rm TT}(\gamma^*\gamma \to \eta_c; W_{\gamma\gamma}, Q^2, 0)\,, \tag{8}$$

here the nucleus photon fluxes for a photon with energy $\omega_A$ are found to be [8,9,10,11,12]

$$\frac{dN}{d\omega_A} = \frac{2Z^2\alpha_{em}}{\pi\omega_A}\left[\xi K_0(\xi)K_1(\xi) - \frac{\xi^2}{2}(K_1^2(\xi)-K_0^2(\xi))\right]. \tag{9}$$

Here $\xi = R_A\omega_A/\gamma_L$, with $R_A = r_0 A^{1/3}$, and $r_0 = 1.1\,$fm is the nuclear radius, $\gamma_L$ is the Lorentz factor. The modified Bessel functions are $K_0$ and $K_1$. We can express the photon-photon center of mass energy $W_{\gamma^*\gamma}$ by transverse momentum of the meson in the final state $p_\perp$:

$$W_{\gamma\gamma} = \sqrt{4\omega_e\omega_A - p_\perp^2}\,, \tag{10}$$

where $p_\perp^2 = \left(1-\frac{\omega_e}{E_e}\right)Q^2$ . Moreover, in an electron-ion cm frame we can find the energies of the photon in terms of the rapidity of the final state:

$$\omega_e = \frac{\sqrt{M^2_{\eta_c}+p^2_\perp}}{2}e^{+y} \quad \text{and} \quad \omega_A = \frac{\sqrt{M^2_{\eta_c}+p^2_\perp}}{2}e^{-y}\,. \tag{11}$$

This brings us to the main ingredient of calculation of the exclusive production, mainly the cross-section of the subprocess $\gamma^*\gamma \to \eta_c$. The standard formulation of the cross-section in case of two virtual photons reads [7,13]:

$$\sigma_{\rm TT}(W_{\gamma\gamma}, Q_1^2, Q_2^2) = \frac{1}{4\sqrt{X}}\frac{M_{\eta_c}\Gamma_{\rm tot}}{(W^2_{\gamma\gamma}-M^2_{\eta_c})^2 + M^2_{\eta_c}\Gamma^2_{\rm tot}}\mathcal{M}^*(++)\mathcal{M}(++)\,, \tag{12}$$

with the kinematic factor $X = (q_1\cdot q_2)^2 - q_1^2 q_2^2$, and the photon virtualities being defined by $Q_i^2 = -q_i^2$. We can use Eq. (5) and the narrow width approximation in Eq. 12, thus we obtain:

$$\sigma_{\rm TT}(W_{\gamma\gamma}, Q^2, 0) \;\;\approx\;\; 8\pi^2\,\delta(W^2_{\gamma\gamma}-M^2_{\eta_c})\frac{\Gamma_{\gamma\gamma}}{M_{\eta_c}}\left(1+\frac{Q^2}{M^2_{\eta_c}}\right)\left(\frac{F(Q^2,0)}{F(0,0)}\right)^2, \tag{13}$$

Wrapping up the formulas above, we can write the total cross-section as follows:

$$\sigma(\gamma^* A \to \eta_c A) = \frac{dN}{d\omega_A}\bigg|_{\omega_A = (M^2_{\eta_c}+p^2_\perp)/(4\omega_e)} 8\pi^2 \frac{1}{4\omega_e}\frac{\Gamma_{\gamma\gamma}}{M_{\eta_c}}\left(1+\frac{Q^2}{M^2_{\eta_c}}\right)\left(\frac{F(Q^2,0)}{F(0,0)}\right)^2. \tag{14}$$

## 3 Numerical analysis and conclusions

In the numerical analysis, we consider energy and target configurations expected to be available at the EIC (BNL)[14,15,16,17,18,19], EicC (China) [20] and LHeC (CERN) [21]. As the future electron-ion collider at BNL will reach luminosities in the $10^{33} - 10^{34}$ cm$^{-2}$s$^{-1}$ range, in our analysis, we assume two ranges to

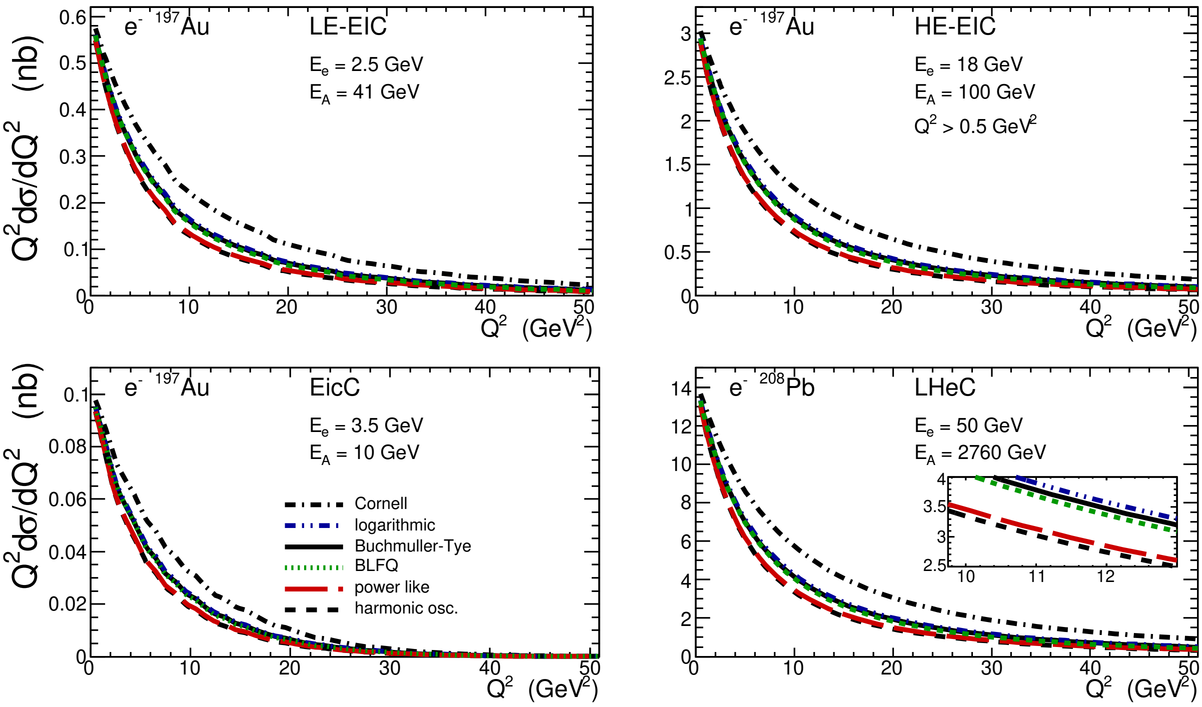


Figure 3 – Differential distribution in photon virtuality multiplied by $Q^2$ for four energy ranges: lower energy range EIC (LE-EIC), higher energy range (HE-EIC), electron-ion collider in China (EicC) and a large hadron-electron collider (LHeC).

be for the electron and $Au$ - ion energies: (a) $(E_e, E_{Au}) = (2.5, 41)$ GeV and (b) $(E_e, E_{Au}) = (18, 100)$ GeV. In Fig. 3 [6], we labelled these two setups as LE - EIC, and HE - EIC, respectively. In the case of EicC predictions, we are concerned with: $E_e = 3.5$ GeV, $E_{Au} = 10$ GeV and $\mathcal{L} = 10^{33}$ cm$^{-2}$s$^{-1}$), and for LHeC: $E_e = 50$ GeV, $E_{Pb} = 2760$ GeV and $\mathcal{L} = 10^{32}$ cm$^{-2}$s$^{-1}$. The electron tagging in the final state could provide $Q^2$ dependence on the differential distribution $Q^2 d\sigma/dQ^2$, see Fig.3 [6]. Our model for the Cornell wave function gives generally larger values. The predictions obtained from the harmonic oscillator and power-like model of the wave function are rather similar to each other in the same manner as results for the BLFQ, Buchmüller-Tye and logarithmic models. The difference between the predictions increases with the center-of-mass energies. The results show that the distribution analysis considered in this letter can be useful for constraining the transition factor[6]. The numerical results for the total cross-section in the range of $50\,\text{GeV}^2 > Q^2 > 0.5\,\text{GeV}^2$ gives approximately 0.1-60 nb.

Our results strongly encourage a more detailed study, which includes realistic experimental limitations for the detectors proposed for installation in the BNL EIC, which we plan to perform in future studies. In principle, analysis can be extended to other final states that the $\gamma^*\gamma$ interaction can produce.

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